

P A P E R S .

- I. *Account of a Determination of the Solar Parallax from Observations of Mars, made at Ascension in 1877.* By DAVID GILL, Her Majesty's Astronomer, Cape of Good Hope.
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CHAPTER I.

HISTORY OF THE EXPEDITION.

THE determination of the Solar Parallax has been well described by the Astronomer Royal as "the noblest problem in Astronomy." The problem presents difficulties so great, and is, at the same time, of such high importance in the progress of Astronomy, that both causes have alike combined to give an increasing interest to each successive attempt to solve it. For the same reasons such attempts have been both numerous and important, and each marks a step in the progress of Practical Astronomy. Every attempt has shown some shortcoming in the previous one, and the result has always been to mark a new era of progress in the science, to create a fresh interest in its pursuit, and to stimulate the invention of more refined instruments and methods.

About a year after the last Transit of *Venus* occurred, when those who had observed the phenomenon began to compare notes, and when attempts were made to select "corresponding phases" in the various records of contact, a doubt began to arise in the minds of astronomers as to whether, after all, we should not again repeat the experience of 1769, and find the observations capable of so many interpretations that an unprejudiced and final discussion would become almost impossible.

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In these circumstances my attention was directed to the opportunity afforded by the approaching opposition of *Mars* (1877, September 5), as a means to obtain an independent determination of the Solar Parallax. The method I proposed, if it had no other merit, had at least the claim that the results would bear no two interpretations, and that—if the observations were conducted on a rigid system and reduced upon a method equally rigid—only one possible result could be obtained from them.

In 1874, in conjunction with Lord LINDSAY, I had already attempted a new method of determining the Solar Parallax. Combining the suggestions of AIRY (*Monthly Notices*, vol. xvii. p. 208), as to the employment of the method of the Diurnal Parallax, with the suggestions of GALLE (*Ast. Nach.* vol. lxxx. p. 1), as to the employment of the Minor Planets, and introducing the Heliometer as the observing instrument, we made at Mauritius an experimental determination of the Solar Parallax by observations of the Minor Planet *Juno*. The details of this investigation, by the liberality of Lord LINDSAY, have been printed and widely circulated as vol. ii. of the *Dun Echt Publications*.

Although the number of observations obtained in this investigation was comparatively small, and the geometrical conditions by no means the most favourable possible, yet the result showed that the method was capable of extreme precision, and that, if carried out on a scale sufficiently complete, a satisfactory determination of the Solar Parallax would probably result.

My attention had long been directed to the Opposition in question by AIRY's paper above quoted; and I was convinced that a very good determination of the Parallax might be made by the method of Extra-Meridian Transits therein advocated. But the very high precision attained by the Heliometer method in the case of *Juno* led me to hope that the same method in the case of *Mars* might lead to a result more accurate than any which could be obtained by the method of Transits. It is true I did not anticipate that it would be possible to measure the angular distance between a star and a disk like that of *Mars* with the same precision as that between a star and a Minor Planet (whose disk is indistinguishable from that of a star), still I believed that a greater precision could in this way be secured than by any other known method of observation.

The method presented the great practical advantages, also possessed by AIRY's method, viz., that no co-operation whatever was required, and that the corresponding evening and morning observations, by which the Parallax would be determined, were to be made *in precisely the same manner, with the same instrument, and by the same observer.*

My first step was to apply to Lord LINDSAY for the loan of his Heliometer, conditionally that I should otherwise obtain the means of observing the approaching Opposition of *Mars*. I shall never forget his kind and ready consent to my proposal, and the effective assistance which he afterwards rendered in every step connected with the progress of the preparations.

In the Autumn of 1876 I applied to the Committee of the Government Grant Fund of the Royal Society for a sum of £500 to enable me to carry out an expedition to St. Helena or the Island of Ascension, and explained fully the objects of the expedition and the proposed methods of observation. I undertook at the same time to defray any costs of the expedition which might exceed £500, at my own charge; or, if the expedition cost less than this sum, to return the balance of the money. There were, however, so many applicants for aid in other important researches, that the Committee did not feel justified in granting so large a sum (one-eighth part of the grant for the year) to one object however important; and believing it of no use to vote a smaller sum, they recommended that the matter should be independently provided for by Government.

To avoid unnecessary delay, however, I referred the matter to the Council of the Royal Astronomical Society. There it received the most cordial support, and finally the requisite sum of £500 was voted by the Council on the understanding that £250 would be repaid to the Society on the joint security of Lord LINDSAY, Mr. DE LA RUE and Mr. SPOTTISWOODE, provided that the latter sum was not obtained from some other source. In 1878, however, the £250 in question was provided by the Government Grant Fund of the Royal Society; but I shall ever bear in grateful remembrance, not only the prompt and timely aid of the Society, but also the generous and ready help of those men by whose aid the last difficulties were removed.

A reference to the following papers, printed in the *Monthly Notices* of the Society, will sufficiently indicate to the reader the general idea on which the Expedition was founded and the manner in which it was carried out.

Monthly Notices, Vol. xxxvii. pp. 310-313.

"	"	"	xxxviii.	"	1-11.
"	"	"	Do.	"	57-58.
"	"	"	Do.	"	89-90.

A popular account of the Expedition, written by my Wife,* who accompanied me on the Expedition and shared throughout its watchings and anxieties, sufficiently describes its incidents and details.

With these references, there only remains for me to make the following brief statement of facts.

The instruments and baggage, excepting the chronometers, were put on board the mail steamer "Balmoral Castle" of the Donald Currie line, then lying in the London Docks. The chronometers were received from the Royal Observatory, Greenwich, on the morning of our departure from London, and were conveyed by train with us to Dartmouth, where we joined the steamer. The following day (15th June, 1877), we sailed from Dartmouth for St. Helena, which we reached on the morning of July 1. While waiting at St. Helena for the return steamer from the Cape of Good Hope to carry us to Ascension, advantage was taken of our ten days' stay in the island to rate the chronometers previous to a chronometric run for difference of longitude between St. Helena and Ascension. On the afternoon of July 10, we left St. Helena, and reached Ascension on the morning of July 13.

The Observatory was at once erected and all was ready for work by July 17. But persistently cloudy weather set in, and none of the intended observations of *Ariadne* could be made.

I was assured that this state of things was quite exceptional; but as the days passed on and only a small amount of work in triangulation of comparison stars could be done at uncertain intervals, I began to see a persistent cause at work. I observed frequently that, whilst we had a band of cloud across the zenith, the horizon to North and South, particularly to South, was often clear. This band of cloud undoubtedly came from Green Mountain (the highest peak on the island and situated near the centre of it to S.E. of Garrison); it seemed to form there and to be carried thence over Garrison and out to sea.

* 'Six Months in Ascension, by Mrs. Gill—An Unscientific Account of a Scientific Expedition.'—John Murray, London. 1878.

The points to be decided were (1st), would it be possible to get south of this cloud ? (2nd), if it were possible, could an Observatory be erected and maintained in such a position ?

Captain PHILLIMORE (the naval officer in command of the island), while warning me of the difficulties attending any change in the site of the Observatory, volunteered in the kindest manner to assist me in every possible way, not only in any change which I might desire to make, but also in search of a site. The night of Monday, July 30, was typical of many that had gone before. That I might not lose any opportunity of observing which might occur, my wife volunteered to make the necessary observations of cloud, which should enable me to decide whether the probabilities of improved sky would compensate for the risks incurred in any proposed change.

Accompanied by two servants, she set off at 10 P.M. and walked as far South as possible, making simultaneous observations of cloud with me at previously arranged instants. These observations were continued from 10 P.M. to 3 A.M. at each half-hour, and from them I was enabled to draw the following conclusions.

(1) When the trade wind is nearly saturated with moisture, the lowering of temperature, produced when the air is raised into higher and colder regions by the mountains, causes a formation of cloud which begins to windward of the peak and passes across the island in the direction of the wind,—always S.E.

(2) The south limit of this stream of cloud passes nearly vertically over the south limit of "South West Plain" about as far as "Round Hill." To clear this cloud it was evident that a point near the south limit of the Island must be occupied, and also pretty far west so that the cloud which may form to windward of Green Mountain shall subtend the smallest possible angle.

Accordingly the following day (July 31) Captain PHILLIMORE and I set out to explore, and to endeavour to find some landing place in the S.W. of the island.

From South Gannet Hill we saw a small bay due south of Saddle Crater, and one of the dry watercourses seemed to render it possible to reach this from the land side. It is impossible to describe the extraordinary character of the lava (locally called clinker), extending for about a mile inland along the southern coast of Ascension, and which, but for this empty watercourse, would have rendered access to the coast all but impossible.

We reached the little bay, and after close examination Captain PHILLIMORE expressed his belief that instruments could be landed there with a smooth sea. At the extreme N.E. corner of the bay there was a narrow strip of sandy beach, and projecting rocks seemed entirely to protect it from the swell from the S.E. The official report in the hands of the Admiralty states that there is *no* landing place here, and the bay is not in sight of any signal station. Captain PHILLIMORE was however convinced of the justness of his conclusions, and undertook the responsibility of conducting the landing; and when my friend Lieutenant (now Commander) HAMMICK, commanding H.M.S. "Cygnet," undertook to give me Kroomen to carry the Heliometer tube by land, I resolved to dismantle the Observatory the following day.

Strangely enough, that same night (July 31) was perfectly cloudless. I obtained a fine set of evening and morning observations of *Mars* and comparison stars, and was strongly tempted by kind advisers to "let well alone," to take my chance of more clear nights like the last in Garrison, rather than run the risk, anxiety, and labour of another transport and erection, as well as the risk of loss of health from want of comforts easily obtained in Garrison, but which could not be obtained in tents under a tropical sun in so desolate a spot. I had, however, already weighed all these chances, and when the men arrived on the morning of August 1, just as I concluded my morning observations of *Mars*, we set to work at once. By the same evening, instruments and houses, together with tents, bedding, furniture, brick, lime, and cement, etc., etc., were all stowed in a large lighter moored near the quay.

At sunrise the following morning (Thursday, August 2) the lighter and some small boats were towed round by a steam tug to our bay, now called "*Mars Bay*." The sea was unusually calm, and under the skilful personal superintendence of Captain PHILLIMORE, everything was landed in perfect safety.

Friday and Saturday were occupied in re-erecting the Observatories and instruments, and on Saturday evening the Heliometer was adjusted and ready for use. I was utterly unable, however, from personal fatigue, to observe that night, but on the following day I obtained good evening and morning observations of *Mars* and comparison stars. During the day of August 5 I found, that without some special arrangements, life would be insupportable at Mars Bay from dust, flies, etc.

Therefore on the morning of the 6th I walked to Garrison in order to

procure carbolic acid, mosquito curtains, and other protections from insects; and also to requisition men to carry sand from the beach, for spreading over the ground in the immediate neighbourhood of the tents in order to keep down the fine volcanic dust which threatened to blind and choke us.

I returned the same night, but had the misfortune to strain a tendon in my knee-joint (an old weakness) in crossing the "clinker," and reached Mars Bay feeling very ill and in considerable pain. The night was fine, but when I attempted to observe I could not see, nor was I able to stand. I sent for a doctor next day. He came at once, and finding that I was knocked up by exposure, anxiety and fatigue, and suffering from a slight local fever called "The Rollers," had me carried across the clinker to Garrison. He promised I should be well in a few days if I placed myself unreservedly in his hands. This I accordingly did, and on Friday the 10th I was able to return to work and to secure morning observations of *Mars*.

From this time observations were continued on every possible opportunity, until October 4, when we went up the mountain for a week, and enjoyed a perfect rest, with the comfortable thought that sufficient observations had been obtained to secure a good result.

From November 12 till December 2, I endeavoured to obtain a series of observations of *Melpomene* for parallax. During this period, with the most unceasing watch, I was able to make only three sets of evening, and one set of morning observations, all of them more or less imperfect. It was now impossible to complete an entirely satisfactory series, and as the early mornings continued cloudy, I applied myself to the completion of the triangulation of the *Mars* stars, to a latitude determination, to observations of the Moon for longitude, and to various other observations which will be described in due course.

On December 24 we returned to Garrison. On the 26th December the instruments, tents, and so forth were brought back to Garrison by sea, and the same night the Transit Instrument was re-erected and time determinations for a chronometric connection of Mars Bay with Garrison were made. Time determinations were also made in Garrison on December 31 and January 8. Meanwhile boxes were being packed, and all was in readiness on January 9, when the "Warwick Castle" arrived; farewells were exchanged, and we sailed for England. At Madeira I had the good fortune to meet Lieutenant Commander DAVIS, U.S. Navy, who was engaged in determining the difference of longitude between Lisbon and Madeira. I obtained comparisons with his

chronometer, the error of which he determined the same night and has since communicated to me.

We reached Plymouth on January 24, and proceeded next day by rail to London.

It now remains for me to express my obligations—

- (1) To Lord LINDSAY, for the loan of his Heliometer (without which the Expedition would have been impossible), and for much valuable aid besides.
- (2) To the Lords of the Admiralty, for the assistance rendered to the Expedition under their Lordships' instructions.
- (3) To the Astronomer-Royal, for the great interest taken by him in the Expedition, for the kind manner in which he represented my probable requirements to the Lords of the Admiralty, and for much valuable help and advice.
- (4) To his Excellency H. R. JÄNISCH, C.M.G., Governor of St. Helena, and to Captain S. P. OLIVER, R.A., for much kindness and assistance during my stay at St. Helena.
- (5) To Captain (now Admiral) PHILLIMORE, to whose energetic and liberal interpretation of his official instructions the success of the Expedition is greatly due; and whose kindly hospitality, joined to that of his wife and family, did so much to brighten our stay on his lonely island. Also to Lieutenant HAMMICK, commanding H.M.S. "Cygnet," for his kind and timely aid to the Expedition.
- (6) To Dr. BUCKLEY, and his medical staff, to Lieutenant CROSS, of the Royal Marines, to Lieutenant SQUIRE, R.N., and the other officers on the island for their ever-ready help and for many acts of kindness during our stay.
- (7) To Mr. DONALD CURRIE for his generous aid in the matter of transport, and for the instructions given by him to the officers of his splendid ships, in consequence of which such care was bestowed that not a single instrument was broken in all the transhipments.
- (8) To Captain HOWSON, of the "Balmoral Castle"; to Captain JONES, of the "Edinburgh Castle"; to Captain WEBSTER, of the "Warwick Castle," and the officers under their command, for their exceeding courtesy and attention during each voyage.

With these remarks, I proceed at once to describe the details of the instruments and methods of observation, the observations themselves, their discussion, and the results.

The original manuscript of this work contains many details which are not given in the following pages.

In condensing this manuscript for publication, I have retained, as far as possible, the full description of methods and instruments ; but, on the suggestion of the Council of the Society, I have omitted the results of Time Determinations, Meteorological Observations, and ancient and modern Meridian Observations of the Comparison Stars, as also the details of the Helio-meter Observations, and of the investigations of the Instrumental Constants. I have also omitted many tables, which in themselves possess no interest, but which would be useful to any one who may desire to examine or check my results.

But I have retained, so far as I know, all facts which are necessary to form a judgment on the accuracy of the work, beyond the stage of preliminary reduction—that is, after the formation of the corrected Instrumental Distances, and after the calculation of the apparent Tabular Distances.

Every portion of the work has been computed in duplicate, and every available independent check has been employed to secure absolute arithmetical accuracy.

The Council of the Society has undertaken to bind the original manuscript and to preserve it for reference in the library of the Society, or for consultation elsewhere with the approval of the Council. In the following pages the reader, who may desire further details, will frequently be referred to the manuscript in question, herein called O.M.

The five volumes, containing the original observations, will also be deposited with the Society. There is also in possession of the Society a copy of the original observations of *Mars* as forwarded from Ascension immediately after the observations were made.

CHAPTER II.

THE INSTRUMENTS.

The Instruments which accompanied the Expedition were as follows :—

1. Lord LINDSAY's Heliometer and its Observatory.
2. A 30-inch Transit by T. COOKE & SONS, York, and a Transit Hut.
3. A Reflecting Circle by TROUGHTON.
4. A Prismatic Circle (10-inch) by WAHNSCHAFTE of Berlin.
5. An Artificial Horizon.
6. Two Barometers and four Thermometers.
7. Six Chronometers, one of them uncompensated.
8. A Chronograph.

1. *The Heliometer.*

This instrument has been minutely described in vol. ii. of Lord LINDSAY'S "Dun Echt Observatory Publications," pp. 1-10.

Before finally packing it for the journey I thought it best to have the instrument sent to London in order that the mounting might be tested with the Polar Axis adjusted for the latitude of Ascension, 8° S. It was a most fortunate circumstance that this precaution was taken. The mounting, though "a universal equatorial" in general construction, had never been intended for use at a lower latitude than Mauritius, and considerable alterations had to be made in the arrangement of the counterpoises, and so forth, before all would work properly in the required conditions. Everything had been nearly completed when an accident occurred which might have been fatal to the Expedition.

The chief alterations had been made, and it came to be a very nice question whether one of the counterpoises would be quite clear of everything in all positions; and before deciding what should be done I went to make a small change in the inclination of the polar axis. A reference to Plate 6

“Dun Echt Publications,” vol. ii., will show how the inclination of the polar axis is changed by the elevating piece S. A plunger fits into this elevating piece; this plunger is hollow, and in it works a screw which is attached to and can be rotated with S. Of course the elevating piece and the plunger used for low latitudes were much longer than those shown in the figure referred to (which is drawn for the latitude of Dun Echt Observatory, $57^{\circ} 9' 6''$), but, by some oversight, the internal screw attached to S was made much shorter than need be. Without my knowing it, the lower end of the polar axis was thus only kept down by a part of a turn of the screw, and the consequence was, that in diminishing slightly the inclination of the axis, the screw gave out, the overhanging weight of the declination axis, heliometer, and counterpoises tilted the box of the polar axis upwards; and, the hour circle not being attached, the polar axis slipped out, and the tube, declination axis, counterpoises, and polar axis came down with a crash upon the floor. Fortunately the heliometer alighted upon the eye-end, which was driven through the floor by the force of the fall and twisted off. As the tube fell, the heliometer head came down upon one of the copper caps provided to protect the slides from dust, and the latter was considerably crushed by the blow; but the fall had been so much broken by the tearing off of the eye-end, and afterwards by the resistance of the copper cap, that the shock was insufficient to bring the cap into contact with the slides, or to strain or damage the *head* in any way. Fortunately also the micrometer was not in its place, and thus, though the apparent damage was very great, no injury whatever was done to the heliometer slides and the micrometer work.

The heliometer tube was sent at once to Messrs. TROUGHTON & SIMMS, whilst Messrs. T. COOKE & SONS, of York, and Mr. JOHN BROWNING, of London, repaired the mounting.

Within ten days all was again in order, the instrument tested, packed, and stowed on board ship.

The only additions to the instrument since it was used in Mauritius were:—

(1) In front of the eye-piece was placed a small prism of reversion, which could be rotated about the axis of the eye-piece,—see fig. 1. The object of this prism was to enable measures of the distance of two objects to be made as if the direction between them was always parallel

with, or at right angles to, the line joining the observer's eyes, and to enable the observer to place a star in the centre of the image of *Mars* without systematic error.

Thus, if I had a tendency to place the image of a star too much to the right of the centre of *Mars*, then by turning the prism round 90° I should see the image of *Mars* reversed, and that of the star, though really reversed, would remain apparently the same as before. I again attempt to place the image of the star in the centre of the planet, and again my natural tendency places the star too much to the right, but the error now produces exactly the opposite effect on the measured distance; hence in the mean the personal error is eliminated.

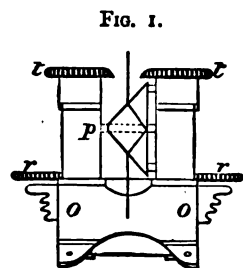


FIG. 1.
p a prism of reversion attached to a mounting *rr* it.

This mounting can be rotated with respect to the ocular *oo* by turning the milled head *rr* which forms part of the mounting.

The same contrivance can also be advantageously employed in measuring double stars. By its use the apparent direction of all double stars can be made to appear the same, relative to the observer's eyes, whatever their true position angle, hour angle, and declination. The correction for personal error in observing position angle is thus reduced to a function of the altitude, and this can be determined by the observer himself from measures of the same double star at different altitudes.

This convenient little apparatus was made under my instructions by Mr. HILGER, of London. I could find no sensible difference in the quality of definition by its use, though there was a very slight loss of light; and I strongly recommend all double star observers to adopt such an apparatus in order to examine their own systematic errors.

(2) In addition to the wire screen G, described in the "Dun Echt Observatory Publications," vol. ii. p. 7, and illustrated in Plate 3 of the same volume, I had three similar spare screens made: *one* with closer wire gauze than G, a *second* with two folds, and a *third* with three folds of wire gauze.

With the triple screen the light of a 4th mag. star was reduced to that of an 8th or 8.5 mag. star; and this screen had to be employed when stars of the 7th or 8th mag. were compared with *Mars*, in order to reduce the brilliancy of the planet sufficiently to permit these stars to be seen upon the disk.

The Observatory.—The Observatory in which the Heliometer was placed had been employed in connection with Lord LINDSAY'S Mauritius Expedition.

A simple octagonal framework of light angle iron carries the wheels upon which the whole Observatory revolves. Into this framework eight uprights of iron gas-tube are fitted, and these uprights support another hexagonal frame of light angle iron. This framework is made rigid by cross-ties of iron wire between the uprights. The roof is a hemispherical framework of angle iron and bent gas-tube. The whole is covered with canvas, excepting an opening, $4\frac{1}{2}$ feet wide, which extends from the ground to about 10° beyond the zenith. The vertical part of this opening forms the door of the Observatory, and is closed by lacing two canvas flaps together. The circular part of the opening is closed by a canvas cover mounted on an iron framework (of radius a little greater than the dome), which is attached to arms working on pivots. These pivots are fixed to the upper octagonal frame, at the opposite extremities of a diameter at right angles to the opening; and, the canvas cover and its mounting being counterpoised, the opening in the roof can be easily closed by drawing down the cover, or opened by throwing it back.

This Observatory was easy to transport, easy to erect, and most convenient in use. The temperature inside was always practically the same as that of the outer air.

2. The Transit Instrument.

This instrument is the *Mathew* Transit belonging to the Society. It was made by T. COOKE and SONS, of York, and its optical qualities are excellent. The object glass is of $2\frac{1}{2}$ inches aperture and 30 inches focal length.

On examining the instrument, with a view to its use on the Expedition, I suggested the following alterations as desirable; and these, with the consent of the Council of the Society, were executed by Mr. SIMMS.

- (1) A new eye-end with micrometer was substituted for the old eye-end, which latter had not any means of measuring the collimation error.
- (2) The cell of the object glass was altered so as to clamp the object glass firmly; formerly it was rather loose.

- (3) The mounting of the level-bubble was altered to **V** bearings, and the whole was made more rigid as well as more convenient in use.
- (4) The pivot **V**s were provided with strong clamping screws by which they could be rigidly fixed when adjusted. Formerly the adjustment depended on a spring pressure against a screw and was not satisfactory.
- (5) A new diagonal eye-piece of higher power than that provided by the original makers was added, as also a Bohnenberger eye-piece for levelling or collimating by reflection from mercury, should this be necessary.
- (6) The bell-metal pivots, before somewhat cut, were reground.

So altered, the instrument was capable of tolerably accurate work.

The Meridian Mark.—For determining collimation Lord LINDSAY lent me two lenses, each of 150 feet focal length. These were at first both used as collimators, one to north, the other to south of the Transit Instrument. In the focus of each lens was placed a small brass upright in which a minute hole was drilled. When these holes were illuminated by a lamp placed behind them, the image of each appeared like a bright sharply defined disk about 4" in diameter, and capable of very accurate bisection. When the Observatory was shifted to Mars Bay, however, owing to the conformation of the surrounding rocks, no site for a mark towards the south could be found. The lens of long focus, however, was mounted on a brick pier, round which, but not touching it, a thick surrounding wall of rough "clinker" blocks was built; and the upper part was covered by a wooden box with circular apertures through which the meridian mark could be viewed. The pier supporting the lens was thus effectually screened from the Sun. The perforated brass plate was similarly mounted, and thus an azimuth mark of considerable stability was secured.

The Reticule and Micrometer.—On arrival at Ascension I found that the webs inserted by Mr. SIMMS were broken. I replaced them by a new reticule of five webs from a fork of spider webs supplied by Mr. SIMMS.

The corrections to the middle wire for an Equatorial star (Head West, Upper Transit) as determined by numerous transits, are—

I	II	IV	V
sec.	sec.	sec.	sec.
+26.817	+14.039	-13.777	-26.102.

In the calculations every observation, on every wire, has been separately reduced to the middle wire employing these intervals. These wires are all mounted on a frame which is moved by the micrometer screw: the head of this screw is divided into 100 parts. By causing the wires successively to bisect the meridian mark, the wire intervals, in terms of the screw, have been determined as follows:—

I	II	IV	V
$\overset{r}{5}^{\cdot}8357$	$\overset{r}{3}^{\cdot}0492$	$\overset{r}{2}^{\cdot}9830$	$\overset{r}{5}^{\cdot}6923$

whence the value of one revolution of the micrometer screw is—

$$\overset{\text{sec.}}{4^{\cdot}5976}$$

In observing slow-moving circumpolar stars the screw has sometimes been employed instead of the fixed wires, the middle web being used to bisect the star.

The Level.—The original level was of the old unsatisfactory form, in which the bubble is cemented into a brass tube. No reliance whatever can be placed on the indications of a level so mounted—for its scale value is liable to change from distortion produced by the different expansions of the brass tube and the glass bubble. At my request Mr. SIMMS remounted the bubble so that it rested like the pivots of a transit on brass Vs. In these Vs it is held by the pressure of brass springs tipped with cork, one cork tip pressing vertically down over each V. There was no cross level, nor was there any adjustment for making the axis of the glass bubble (when the level rested on the pivots) to coincide with a plane which should at the same time pass through the axis of rotation of the Transit.

Thus, when the level was placed on the pivots, the indications were different when the level was rotated on the pivots a little towards north or south.

I altered the Vs by careful filing and scraping, till at last this adjustment was entirely satisfactory, and so, that not the slightest difference of reading could be detected when the level was inclined as far to north or south as it would stand without support.

The levellings became in consequence most satisfactory and consistent throughout.

The value of one division of the level was first determined as follows.

The Transit was provided with a cap or cover to the object glass having a small circular aperture for observing the Sun. This cap I placed on the pier carrying the meridian mark, care being taken to place it truly at right angles to the axis of the collimation lens. A piece of white card inclined at an angle of 45° behind this aperture made a satisfactory illumination, so that the hole appeared as a sharply defined white disk in the field of the Transit.

The horizontal diameter of the image of this disk, as seen in the Transit, was then measured in terms of the micrometer screw.

The level was then placed on the tube of the Transit Instrument, one leg of the level resting on one cone of the tube, the other leg upon the other cone. Obviously it was easy now, by shifting the level towards the object-glass end or towards the eye end, to change the inclination of the axis of the level relative to the optical axis of the Transit. The horizontal web of the Transit Instrument was then made tangential to the lower limb of the white disk, and the level was shifted along the cones till the bubble read near one end of the scale. Without touching the level, the slow-motion screw was employed to move the Transit till the horizontal wire was tangential to the upper limb of the white disk. The inclination of the Transit was thus changed by an amount equal to the angular diameter of the white disk; hence the difference of the level readings corresponded with this change.

To eliminate the effects of ellipticity of the hole, the measures were repeated after the cap had been turned 90° , so that the diameter which in the former case was vertical became now horizontal. In this way the diameter of the disk was found.

In terms of the micrometer screw	= 1.767 rev.
In terms of divisions of the level	= 41.60 divisions

$$\text{Hence the value of 1 division of the level} = \overset{\text{sec.}}{0.195}.$$

This determination was made in Garrison.

Afterwards at Mars Bay, when the micrometer had been turned round 90° for the purpose of using the Transit as an equal-altitude instrument, the level value was determined in a similar manner, the only difference being

that the change of inclination of the Transit axis could now be measured directly by the micrometer screw.

In this way, the following results were obtained at Mars Bay, December 9, 1877:—

Divs. of Level.	Rev. of Micr.		rev.	sec.
16.24 = 0.653	}	∴ 1 d. of level =	0.0410 =	0.188
15.77 = 0.658				
6.80 = 0.293	}	∴ 1 d. of level =	.0434 =	0.200
6.79 = 0.297				
3.80 = 0.161	}	∴ 1 d. of level =	.0419 =	0.193.
4.83 = 0.196				
4.82 = 0.206				

These determinations appeared to show that the level indications, though not quite uniform in value, were yet quite sufficiently so, and that the value of one division of the level (viz. 0.195) adopted in the preliminary calculations of the Transit observations, was quite sufficiently exact for practical purposes.

The Stand.—The Transit Stand consists of two strong cast-iron pillars attached to a cast-iron base.

The pivots rest on V-shaped bearings both movable by screws, opposed by springs, one screw giving adjustment in level, the other in azimuth. Powerful clamping screws were introduced in both these adjustments by Mr. SIMMS, as he had found by experience that no permanence of adjustment could be secured without some such arrangement.

The slides in which the Vs move are carried by brass caps, which fit upon the turned ends of the cast-iron pillars; but this mode of attachment was not found to be very satisfactory. The caps had probably been fitted at a much lower temperature than we experienced at Ascension; at all events, one of them was slightly loose, and the retaining screws were neither powerful nor well screwed up. In consequence of this misfit there appeared to be a tendency of the instrument to assume one of two azimuths differing 5" or 6" of arc. I detected this source of error when the instrument was erected in Garrison, the detection there being easy because there was a

meridian mark both to north and south of the instrument. Change of azimuth in reversal was therefore eliminated in the mean of the *apparent* collimation errors derived from the two collimators, and half the difference of the two *apparent* collimation errors gave the true change of azimuth.

Having discovered the error, I was able to remedy it by inserting a thin slip of tissue paper to tighten the fit of the cap, and forcing the screws well home.

The Pivots.—These are of bell-metal. As I received the instrument from the Society the pivots were a good deal cut, but, as before stated, they were reground by Mr. SIMMS. They then appeared to be sufficiently perfect as to their cylindrical form, the level readings being sensibly the same when the telescope was directed to different and opposite zenith distances.

The mean of a long and careful series of levels in frequently reversed positions gave—

$$\begin{array}{c} \text{d.} \\ +0.64 \end{array}$$

for the actual difference of readings for Micrometer Head West—East. The **Vs** were accurately rectangular; hence the corrections to be applied to the actual readings in one position of the axis to find the true inclination of the axis were—

$$\begin{array}{rcl} & \text{d.} & \\ \text{For Micr. Head W.} & - & 0.16 \\ \text{,,} & \text{E.} & + 0.16. \end{array}$$

and these corrections have been applied to all the observations.

The Transit Hut.—This was an excellent wooden hut, lent by the Lords Commissioners of the Admiralty at the suggestion of the Astronomer Royal.

It was originally made at Greenwich (on a plan of Sir GEORGE AIRY'S) for the Transit of *Venus* Expedition, and had been used at Mokattam, near Cairo, by the British Transit of *Venus* party under Captain ORDE BROWNE. It was built in very few parts, fitting together by screw bolts, so that it could be erected or dismantled with great facility. At Mars Bay, besides the brick pillar, insulated from the floor, upon which the Transit Instrument was mounted, it contained the Barometer used in the observations, the Chronometers, and a small iron bed; for, until the sleeping tent was floored with wood, the Transit Hut was by far the most comfortable bedroom.

3 and 4. The Reflecting Circles.

The Instrument No. 3 is the *Lee Circle*, by TROUGHTON, belonging to the Society.

I borrowed it with the intention of employing it for the determination of time at St. Helena, and also at Ascension until the Transit Instrument should be erected. The instrument is an admirable one, its optical qualities are very high, and the divisions are exquisite. I found the Instrument, however, rather heavy for use by hand, and I had no time to familiarise myself with the use of the Stand. Throughout the observations therefore (excepting the first day's observations at St. Helena), I used the Instrument No. 4.

The latter is an admirable Prismatic Circle of 10 inches diameter, on the well known model of PISTOR and MARTINS, but made in this instance by WAHNSCHAFTE, of Berlin. I saw it amongst the collection of Instruments purchased from the Loan Collection at South Kensington, and on application to the authorities it was kindly placed at my disposal for the Expedition. As to optical definition, excellence of graduation, and general convenience in use, this instrument leaves little to be desired.

5. The Artificial Horizon.

This was of the usual modern construction—by ELLIOTT, of London, and was kindly lent to me for the Expedition by Lord LINDSAY. In use the observations were made with alternately reversed position of the cover, in order to eliminate the effects of possible want of parallelism in the glass windows. There appeared, however, to be no sensible errors due to the cause in question.

6. The Meteorological Instruments.

These were supplied through the kindness of Mr. ROBERT H. SCOTT, F.R.S., from the Meteorological Office.

Of the two Barometers and six Thermometers supplied, only one Barometer and one Thermometer were used throughout the Expedition, for purposes connected with Astronomy.

The Barometer was of the form called the Marine Barometer. The tube

is much contracted at the bottom to prevent "pumping" at sea, and in consequence of this the reading is apt to lag behind somewhat, that is to say with a rising Barometer the reading will generally be too low, and with a falling Barometer too high. From an extended series of experiments on similar Barometers (which I have since made in comparing the Cape Standard Barometer with the Kew Standard through the medium of Marine Barometers transported between Kew Observatory and the Royal Observatory, Cape of Good Hope), I find that the probable error of the indications due to this cause, including errors of reading, is $\pm 0^{\text{in}}.002$, and that the greatest error that has occurred amounts to $\pm 0^{\text{in}}.010$, and this only on 2 or 3 comparisons amongst some hundreds.

Even this exceptional error would have quite an insensible influence on any of the Refraction corrections with which we have to deal in the following pages.

The corrections of the Barometer, as determined at Kew, are as follow.

Marine Barometer Adie B.T. 422.

Examined at Kew, May 1877.

	At	in. 27.5	in. 28.0	in. 28.5	in. 29.0	in. 29.5	in. 30.0	in. 30.5
Correction to Kew Standard		+0.002	+0.001	+0.001	± 0.000	± 0.000	-0.001	-0.001.

In February 1878, the Barometer was compared at the Meteorological Office with the result—

Correction to Kew Standard	At	in. 30.0
		± 0.000 .

At the same time the correction to the attached Thermometer was found to be—

$$-0.1^{\circ} \text{ F.}$$

It is obvious that for all purposes of refraction this Barometer may be considered perfect, and accordingly no index correction was applied in any of the computations.

The Thermometer used for determining the temperature of the air was mounted in a well-ventilated wooden crib attached to the southern front of

Thermometer Negretti and Zambra B.T. 1733.

Examined at Kew, May 1877.

	At	32°	42°	52°	62°	72°	82°	92° F.
Correction for reduction to Kew Standard	}	0.0	0.0	0.0	+0.1	0.0	0.0	0.0.

At 51° F.
-0.5.

Correction for reduction to Kew Standard

7. *The Chronometers.*

(A) A mean time Chronometer lent by Lord LINDSAY.
(B) Do. do. lent by Mr. HARTNUP, Liverpool.
(C) A sidereal Chronometer belonging to myself.
(D) An uncompensated Chronometer, lent by Lord LINDSAY. This Chronometer was employed as a chronometric thermometer. It is the instrument which was employed for a similar purpose on Lord LINDSAY's Mauritius expedition.

(E) A mean time Chronometer, lent by Lord LINDSAY.

(F) A sidereal Chronometer, lent by Lord LINDSAY. This last Chronometer is fitted with an electric-break spring, and can be used with a chronograph. It has been used throughout as the observing Chronometer in the observations of *Mars*, and in all the observations at Ascension, except in observations of altitudes of the Sun with the reflecting circle, and on July 23, when Chronometer C was used, because Chronometer F had stopped. On examination I found that the thin silk-covered copper wire connected with the electric contact had become entangled with the mainspring barrel and wound round between it and the plate till the Chronometer stopped. I took the Chronometer partly to pieces, removed the barrel, and put all again in order, so that the instrument was ready for use on the evening of the 24th.*

All these Chronometers were fitted with their gimbals into one long box divided into six compartments. This box was suspended by strong india-rubber rings inside another box. Thus the Chronometers were protected, not only from sudden changes of temperature, but also from injury by shocks in transport. At sea, of course, the Chronometers were allowed to swing freely, in transport by land the gimbals were clamped.

Excepting Chronometer E, all the instruments were tested at Liverpool by the kindness of Mr. HARTNUP. The results are given in the following Table. The sign + indicates a losing rate.

* The rate of this chronometer was remarkably satisfactory. The greatest deviation from its mean daily rate during August and September was $0^{\circ}24$, and the probable daily variation of its rate—computed from the differences from mean daily rate ($+1^{\circ}32$)—amounted only to $\pm 0^{\circ}11$, which includes the error of the time determination besides the variation of the chronometer rate.

		Temperature 55° F.	70° F.	85° F.
Chronometer A	May 14 to May 19	^s	^s + 2'63	^s
	„ 21 „ „ 26	+ 0'68		
	„ 28 „ June 2		+ 1'27	
	June 4 „ „ 9			+ 2'65
Chronometer B	April 9 to April 14			+ 2'75
	„ 16 „ „ 21		+ 3'53	
	„ 23 „ „ 28	+ 4'63		
	„ 30 „ May 5		+ 3'37	
	May 7 „ „ 12			+ 2'85
	„ 14 „ „ 19		+ 3'95	
	„ 21 „ „ 26	+ 5'70		
	„ 28 „ June 2		+ 3'72	
Chronometer C	June 4 „ „ 9			+ 2'92
	March 26 to March 31	- 3'17		
	April 2 „ April 7		- 4'17	
	„ 9 „ „ 14			- 3'42
	„ 16 „ „ 21		- 4'03	
	„ 23 „ „ 28	- 3'43		
	„ 30 „ May 5		- 4'30	
	May 7 „ „ 12			- 4'03
	„ 14 „ „ 19		- 4'55	
	„ 21 „ „ 26	- 4'38		
Chronometer D	„ 28 „ June 2		- 4'53	
	June 4 „ „ 9			- 4'20
	May 14 to May 19	- 26'78	+ 60'45	
	„ 21 „ „ 26			
Chronometer E	„ 28 „ June 2		+ 61'55	
	June 4 „ „ 9			+ 144'97
	May 14 to May 19		- 1'42	
	„ 21 „ „ 26	- 0'87		
Chronometer F	„ 28 „ June 2		- 1'57	
	June 4 „ „ 9			- 1'27

On the supposition that the rate can be represented by the expression

$$R + c (T - t)^2 + na$$

where

T is the temperature at which the chronometer goes fastest.

R the daily rate at that temperature for the mean epoch.

n the number of days from mean epoch.

a the daily acceleration of rate.

t the temperature for which the rate is required,

c the temperature coefficient.

the following values of c and T have been computed by HARTNUP's formulæ

Chronometer A	c . Rate very irregular.	T .
B	+·0016	94°
C	+·0024	72°
F	+·0019	74°.

The rate of the Chronometer A was so irregular that I could not obtain satisfactory coefficients from it, and I have not employed the instrument in any of the determinations. The instrument, however, in itself I know to be excellent, and I believe the irregularities are due entirely to bad oil.

For the Chronometric Thermometer D, I have assumed the rate to vary directly as the temperature, and I find the expression for its rate to be

$$61^{\text{sec.}} \cdot 0 + 5 \cdot 70 (t - 70^\circ).$$

With the temperature corrections for the Chronometer E, I shall deal further on.* The methods of reduction will be fully detailed in Lord LINDSAY'S *Dun Echt Publications*, vol. iii.

8. *The Chronograph.*

This was a very convenient instrument kindly lent by Lord LINDSAY. It had been used in the Mauritius Expedition by Dr. COPELAND with admirable results, and I had intended to employ it in observing occultations and Moon

* See O. M. chap. iv.

culminations. I found, however, that the additional labour involved in preparing it for work, keeping it in order, and afterwards reading off the observations was more than I could safely undertake. The instrument, therefore, was not employed, the observations for which it was intended having been made by the eye and ear method instead.

CHAPTER III.

DETERMINATION OF THE ERRORS OF THE CHRONOMETERS.

At St. Helena, and on arrival at Ascension before the erection of the Transit Instrument, the error of one chronometer was determined by observations of altitudes of the Sun with one of the Reflecting Circles.

I had not much previous experience in the use of portable reflecting instruments, and accordingly such accuracy has not been attained as is usual in the case of experienced observers with like instruments. Every precaution was, however, taken to avoid systematic errors, and I think that the mean results possess all sufficient accuracy.* The index error was determined before and after each set of observations by repeated opposite measures of the Sun's diameter. Both limbs of the Sun were observed, and, except on the afternoon of July 2, when clouds prevented the continuance of the observations, the cover of the artificial horizon was reversed between each pair of sights. In the case of the Wahnschaffe Circle the Sun-shades were also reversed at the same time. The Barometer and Thermometer were read before and after the observations, and the refraction was rigidly computed by Bessel's Tables.

Tables of the separate results so observed will be found in the O.M. Chap. III.

* The probable error of a time-determination in no case amounts to ± 0.2 , and is generally much less.

Method of Observation with the Transit Instrument.

The time determinations at Ascension after July 14 were made by means of the Transit Instrument.

In the preceding chapter I have shown that the instrument had a tendency to change its azimuth on reversal. I therefore made it a rule before observing to secure a satisfactory determination of collimation in the first place, by reversal and re-reversal, till I had succeeded in getting satisfactory repetitions of the same micrometer readings on the meridian mark on re-reversal. The reading for geometric collimation was soon found to be very constant. Observations, including polar stars or stars of sufficiently opposite zenith distance to give both chronometer error and azimuth, were then made without reversing the instrument. When the instrument was reversed in the same evening the azimuth was either determined again, or the change of azimuth was determined by reference to the meridian mark ; many of the time determinations were made during the day, to save the observer's time at night ; for this purpose bright stars culminating on opposite sides of the zenith were usually observed. In case of a very few stars which culminated near the zenith, such as β *Libræ*, α *Libræ*, and α *Virginis*, an observation of a single star was sometimes considered sufficient for a time determination, the azimuth being deduced with all necessary accuracy by reference to the approximately known azimuth of the meridian mark.

The level error was determined, in the usual way, by use of the spirit-level ; four readings of the level in alternately reversed positions were usually made before and after each set of observations for time. The correction for difference of diameter of the pivots was always applied.

Determination of the Errors of the Stationary Chronometers.

Immediately before or after (sometimes both before and after) each time determination, the observing chronometer was compared with the stationary chronometers, and from these comparisons the error of each chronometer was deduced. The results will be found in the O.M. Chap. III. Table IV. The extremely regular rate of the Chronometer (F) employed in the Heliometer Observations, shows that no case of tripping or of sudden loss or gain occurred throughout the observations.

Deduction of the Error of Chronometer F for Epochs intermediate between Time Determinations.

The fatigue consequent upon the long-sustained Heliometer Observations made it impossible for a single observer to determine time every night, and recourse was had to the stationary chronometers to carry on the time.

Before and after each series of Heliometer Observations, the observing Chronometer F was compared with each of the stationary chronometers. Then, regarding the error of each stationary chronometer as known by interpolation from the preceding and following determinations of its absolute error, the error of Chronometer F was found from each stationary chronometer. The results were quite satisfactory. Thus, for example, though no time determinations were secured between Sept. 14 and Sept. 22, we find that in the intermediate comparisons the greatest difference between the errors of Chronometer F as determined by each of the four chronometers occurs on Sept. 18, as follows:

	Chronometer F Slow.	Dif. from Mean.
Predicted Correction by Chronometer B	h m s +0 1 26.54	s +0.24
" " " C	25.97	- .33
" " " E	26.56	+ .26
" " by its own rate F	26.11	- .19
Mean ...	+0 1 26.30	...

In general the agreement is much closer than this.

At the time of opposition, when the apparent motion of the planet is most rapid, the change of apparent place barely exceeds 0".01 of arc in a second of time. In the most unfavourable case (above quoted) the probable error of the time determination does not amount to one-fifth of a second of time; we may therefore dismiss all further consideration of the effects of errors in the time determinations upon the resulting place of the planet.

Tables of the separate time determinations and the results derived from the daily inter-comparisons of the chronometers are, however, given in the O.M. Chap. III. Table V.

CHAPTER IV.

DETERMINATION OF LATITUDE AND LONGITUDE OF THE OBSERVING STATIONS.

Latitude of Station at St. Helena.

The care with which the latitude of the Observatory at St. Helena has been determined rendered it quite unnecessary for me to make any attempt to confirm the value found by JOHNSON, viz. $15^{\circ} 55' 26''$. My observations at St. Helena were made on the stone platform in front of the Observatory, and JOHNSON'S latitude was used in their reduction.

Latitude of Station in Garrison—Ascension.

Time did not permit me to determine the latitude of the temporary Observing Station (Garrison) in Ascension, and, in the reductions of the observations, I have adopted the latitude determined by the Surveying Officers of the Admiralty for the Observing Pillar in Garrison. No observations were made in Garrison which demand for their reduction a very precise value of the latitude—the value adopted had only to be sufficiently exact for computing the refraction in case of the triangulation of the *Mars* comparison stars, and the refraction and parallax for the observations of *Mars* made on July 31—after which the Observatory was removed to Mars Bay.

The latitude of the Observing Pillar in Garrison adopted is $-7^{\circ} 55' 50''$.

Latitude of Mars Bay.

For the determination of this latitude I had originally intended to rely upon observations with the reflecting circles.

It afterwards occurred to me that a very good substitute for an equal altitude instrument might be made by turning the micrometer of the transit instrument round 90° , and employing the level attached to the small setting circle on the axis to direct the tube to equal altitudes, when by simple reversion of the pivots on the **V**s the telescope was directed to opposite zenith distances.

The practical difficulties were (1) that the level attached to the setting circle was so far from sensitive * ($1 \text{ div}^n = 8''\cdot61$) that a very high accuracy could not be looked for ; and (2) the way in which the level was mounted did not permit of another bubble being easily applied. I attempted in fact to substitute the bubble of the axis level for the small setting level bubble, but I had no means of fixing it except by tying with string and indiarubber bands. I could not make an attachment so rigid that it was not liable to shift on reversal of the instrument, by the mere shock of setting the pivots down on their bearings. The results obtained with the level attached to the setting circle were as follows :

1877, Dec. 8.					Latitude of Transit Hut, Mars Bay.
By Bradley	225	N and χ Phoen. S ...	...	...	$-7^{\circ} 59' 14''\cdot48$
21 Arietis		N and ϵ Eridani S	...	...	$14''\cdot72$
84 Tauri		N and ν , Eridani S	...	...	$17''\cdot83$
With a probable error ...					$-7^{\circ} 59' 15''\cdot67$ $\pm 1''\cdot50$

On the last night at Mars Bay the roof was removed from the transit hut, and observations were made, and afterwards reduced by BESSEL'S method, as described by CHAUVENET, *Spherical Astronomy*, pp. 268-271.

The result of six pairs of stars so observed and reduced gave for the latitude of Transit Hut, Mars Bay,

$$-7^{\circ} 59' 15''\cdot81.$$

with a probable error $\pm 0''\cdot50$.

The difference of longitude between JOHNSON'S Observatory, St. Helena, and the Observatory Station in Garrison was determined by chronometers, the time determinations at both stations being made with the reflecting circle.

* The bubble also, I believe, was not ground to a regular curve ; the scale was drawn by myself on paper, and pasted upon the bubble. The same part of the bubble was used throughout on Dec. 8 ; thus eliminating as far as possible the errors due to the level.

Observing Pillar in Garrison, Ascension, W. of JOHNSON'S Observatory, St. Helena.

	Without Corrections for Temperature.	With Corrections for Temperature.
By Chronometer B	$\begin{matrix} h & m & s \\ +0 & 34 & 46.61 \end{matrix}$	$\begin{matrix} h & m & s \\ +0 & 34 & 46.19 \end{matrix}$
„ C	48.04	47.89
„ E	41.94	43.96
„ F	46.05	45.91
Mean	$+0 \ 34 \ 45.66$	$+0 \ 34 \ 45.86$
Probable error ...	$\pm \ 1.02$	$\pm \ 0.61$

The application of the temperature corrections thus diminishes the probable error to nearly one-half its former amount, and it is exceedingly improbable that the result can be 1^s in error. The other determinations which I have been able to find are—

			$\begin{matrix} h & m & s \\ +0 & 34 & 46.87 \end{matrix}$
Duperry	...	...	
Foster	...	...	48.53
Fitzroy	...	...	45.67

The indiscriminate mean of these gives $0^h \ 34^m \ 47.0^s$, but as I have not been able to find any data by which to attach weight to these determinations, I have thought it best not to introduce any of them into the result, especially as the most modern, and presumably the best, is almost in exact agreement with my own result.

Longitude of Mars Bay from JOHNSON'S Observations of Moon-Culminations at St. Helena.

JOHNSON, from observations of corresponding Moon culminations at St. Helena, Greenwich, Cambridge, and Cape of Good Hope, during the years 1830, 1831, 1832, and 1833, deduced for the longitude of his Observatory at St. Helena—

$\begin{matrix} h & m & s \\ 0 & 22 & 54.6 \end{matrix}$ W. of Greenwich.

This result rests practically on eight observations only, for there are only four observations of the second limb, and JOHNSON has very properly taken the mean of the independent results from the first and second limb.

JOHNSON also observed the Solar Eclipse of 1832, July 27, and deduced a somewhat smaller longitude from it; but he remarks, 'for the present I assume the longitude of the Observatory at St. Helena $0^h 22^m 54^s.6$ W. of Greenwich, though I think it not improbable that future observations will point out a diminution two or three seconds to be necessary.'

Taking JOHNSON's longitude, however, for the present, we have—

St. Helena (Johnson's Observatory), W. of Greenwich	...	h	m	s
Ascension (Transit Hut in Garrison), W. of St. Helena	...	+	0 22	54.60
Ascension (Transit Hut in Garrison), W. of Greenwich	...	+	0 57	40.46
„ Magnetic Observatory, W. of Transit Hut	...	—		0.37
„ Mars Bay, W. of Magnetic Observatory	...	—		3.29
Mars Bay, W. of Greenwich, by Johnson's determination of				
Longitude of St. Helena		+	0 57	36.80

Longitude of Mars Bay, from Occultations.

At Mars Bay I had three satisfactory observations of occultations of stars by the dark limb of the Moon, viz.—

1877, Sept. 14	B.A.C. 6063	Disappearance at	h	m	s	by Chronometer F.
18	31 Capric.	„	18	20	33.1	„
Oct. 3	ρ Leonis	Reappearance at	5	45	29.5	„

Employing the errors and rates of Chronometer F. from O.M. Chapter III. Table V., we obtain the following corresponding instants of local sidereal time :

B.A.C. 6063	Disappearance, Sept. 14 at	h	m	s	Local Sidereal Time.
31 Capric.	„	18	18	21	59.39
ρ Leonis	Reappearance Oct. 3	5	47	14.98	„

The places of the stars of comparison have been derived as follows :

	No. of Obs.	R.A. 1877 ^o .	Adopted Weight.	No. of Obs.	Dec. 1877 ^o .	Adopted Weight.
<i>B.A.C. 6063 (Lacaille 7508).*</i>						
Cape Cat. 1850 † ...	2	^h 17 ^m 48 ^s 55.87	1	2	−28° 2' 35.5"	1
Washington Cat. 1860	10	55.81	2	9	36.4	2
Greenwich Cat. 1872 ...	3	55.92	2	5	36.1‡	2
Greenwich Obs. 1877 ...	1	55.88	1	1	36.2	1
Cape Obs. 1878 ...	2	55.85	2	2	36.1	2
Mean ...		17 48 55.86			−28 2 36.2	
<i>31 Capricorni. §</i>						
Armagh 1835 ...	6	21 11 22.60	0	^I 1853	−17 58 35.4	0
Greenwich Cat. 1864 ...	3	22.59	1	3	37.4	1
Greenwich Obs. 1877 ...	5	22.56	2	4	37.6	1
Mean ...		21 11 22.57			−17 58 37.5	
<i>ρ Leonis.</i>						
Greenwich Cat. 1872 ...	40	10 26 20.02	1	24	+9 56 19.8‡	1

The adopted apparent places of the stars are derived from these mean places, and are as follows:

* Reduced without proper motion. The N.A. place is very much in error.

† From the Cape Catalogue for 1850, now in course of preparation for press.

‡ With corrections, given in the introduction of the Catalogue for latitude and refraction, applied.

§ The proper motion of the Greenwich Catalogue seems to be erroneous. The above places of 31 Capricorni, reduced without proper motion, appear to show that the proper motion is insensible in both coordinates.

|| A very discordant observation (August 29) is rejected; the correction for latitude and refraction mentioned in the Introduction to the *Greenwich Observations* is applied.

1877.	App. R.A.	App. Dec.
Sept. 14, B.A.C. 6063	$\begin{smallmatrix} h & m & s \\ 17 & 48 & 59\cdot31 \end{smallmatrix}$	$-28^{\circ} 2' 47''\cdot2$
18, 31 Capric.	$21\ 11\ 26\cdot73$	$-17\ 58\ 25\cdot9$
Oct. 3, ρ Leonis	$10\ 26\ 21\cdot98$	$+9\ 56\ 12\cdot1$

I have been unable to obtain a sufficient number of meridian observations of the Moon, about the epochs of these occultations, to deduce from them satisfactory corrections of HANSEN'S Tables. I had therefore recourse to NEWCOMB'S corrections to HANSEN'S Tables,* which give for the epochs in question the following values :

1877.	$\Delta\alpha$	$\Delta\delta$
d	^B	
Sept. 14 ^h 49 G.M.T....	— 0 ^h 74	— 0 ^h 3
18 ^h 31	— 0 ^h 55	— 3 ^h 1
Oct. 3 ^h 75	— 0 ^h 65	+ 4 ^h 9

In forming the equations the places of the moon have been taken from the *Nautical Almanac*, and to these places the above corrections have been added. The parallax and semi-diameter are taken from the same authority.

The resulting equations are—

[illegible]

* *Investigation of Corrections to Hansen's Tables of the Moon; with tables for their application*; by Simon Newcomb, Professor U.S.N. Forming Part III. of Papers published by the Commission of the Transit of Venus. Washington, 1876.

$\Delta\pi_0$ the correction required to HANSEN's Constant of the Lunar Parallax.

$\Delta\alpha$ and $\Delta\delta$ are the corrections required in Right Ascension and Declination to *Hansen's Tables with Newcomb's Corrections added*, expressed in seconds of arc.

The definitive longitude which will be arrived at from these equations will depend a good deal upon what semi-diameter of the Moon is finally adopted.

If we adopt the values deduced by Neison (*Monthly Notices*, vol. xxxiv. p. 358), viz.

Correction of the Semi-diameter adopted by the Astronomer Royal (15' 34".68).

For Disappearance of Stars at the dark limb	...	...	-2".74
Reappearance	"	"	-1.08*

these values become for HANSEN's Tables

Correction of Hansen's Semi-diameter.

For Disappearance of Stars at the dark limb ...	...	-1".53
Reappearance " " ...	...	+0".13.

we obtain

Longitude of Mars Bay, W. of Greenwich.

						h	m	s
By Disappearance of	B.A.C. 6063	...	...	...	...	+0	57	35.31
"	31 Capricorni	...	...	...	...			40.41
By Reappearance of	ρ Leonis	...	...	...	...			42.44
Mean	...	...	...	...	...	+0	57	39.38.

* I adopt Mr. NEISON's results with the more confidence that they are remarkably confirmed by an unpublished investigation of Mr. W. H. FINLAY, first assistant at the Royal Observatory, Cape of Good Hope. These results I quote with his permission :—

From 12 *good* observations of Disappearance at the dark limb.

Correction of Astronomer Royal's Semi-diameter ... $-2''.54$

From 10 *good* observations of Reappearance at dark limb ... $-1''16$.

These results are obtained from occultations observed at the Cape of Good Hope.

If Mr. STONE'S correction of HANSEN'S parallax is employed, the final result is diminished by 0^s.2, and the accord of the observations is rather better. The chief uncertainty, however, still remains, viz., that of the true correction to be applied to the geocentric R.A. and Dec., and from this cause the result is probably uncertain to two or three seconds of time.

In round numbers we have

Longitude of Mars Bay.

			h	m	s
From Johnson's Moon Culminations at St. Helena and					
Chronometric connection with Ascension	...		+0	57	38
From Occultations observed at Mars Bay	...	...	+0	57	39.

In the following reductions the geographical latitudes and longitudes adopted are

			Latitude.		Longitude.
					h m s
Garrison, Ascension	...	...	-7° 55' 50"	W. 0	57 42
Mars Bay	...	...	-7 59 15	W. 0	57 39.

and these coordinates are amply exact enough for our purposes.

The reader is referred to the O.M. for details of the latitude and longitude determinations.

CHAPTER V.

THE MERIDIAN OBSERVATIONS OF COMPARISON STARS.

Before leaving England for Ascension I wrote to the Directors of the principal Observatories throughout the world, requesting meridian observations of the comparison stars which I intended to employ in the observations of *Mars*.

In answer to my appeal I received, in all, 15 series of observations, and I desire now to record my grateful thanks to the Directors of the respective Observatories for their kind and hearty co-operation.

Of these 15 series two were not employed. The meridian series made at Dublin, owing to the illness of the first assistant, was not sufficiently complete, and a very fine extra-meridian series made by Mr. HARTNUP at Liverpool I have not been able to employ. Though the observations at Liverpool are most admirably accordant *inter se*, the results are affected by a small, though evident, systematic error, which, from the employment of the instrument in one position only, I could not eliminate.

It has not been considered necessary to reprint these observations ; the reader is referred to the O.M., where they are given together with all the known older observations employed to determine the proper motions.

CHAPTER VI.

THE HELIOMETER OBSERVATIONS—CORRECTION AND COMPUTATION OF INSTRUMENTAL RESULTS.

The methods of observation which I have employed in the use of the Helio-meter at Ascension, as well as the methods employed in the after reduction of the measures, are very similar to those which I adopted in the case of the *Juno* observations at Mauritius—described, with considerable detail, in vol. ii. of the *Dun Echt Publications*. There are some important points, however, in which the present observations and reductions differ from the latter series, and to these I must refer.

1. The errors of the micrometer screw have been investigated with much greater accuracy, and separately for both scales.
2. It has not been considered necessary to re-investigate the division errors of the scale ; indeed, without special appliances, which I did not possess, I think it would hardly be possible to determine the errors in question with more accuracy than they have already been determined, by the methods described in Chapter II. of the *Dun Echt Observatory Publications*.
3. It is shown, in connection with the investigation of the errors of the screw, that the *error of run* of the micrometer is remarkably constant, the probable error of its determination generally exceeding the apparent changes.

The establishment of this fact (see *investigation of runs* in the present Chapter) permits the employment of a much simpler method of reduction than that used in the case of the *Juno* observations ; it permits, in fact, the correction for error of run to be incorporated in the table of screw corrections.

4. In the *Juno* observations the division preceding as well as that following the zero of the microscope was read. It is decidedly an advantage, in some respects, to observe in this way; but in the case of *Mars* it was certain that the placing of the image of the star on the limb, or in the centre of the disk, would be subject to much greater probable error than the bisection of the scale divisions with the micrometer. I therefore thought it better to devote the limited time, when the conditions of parallax and definition together were most favourable, to the securing of numerous bisections of the planet with single division-readings of the scale, rather than have fewer bisections of the planet and scale readings of double weight. In observations of stars, where it was no object to complete a set of measures within a given limited time, and since also a higher accuracy in the star bisections could be attained, the system of reading the division both preceding and following the micrometer zero was employed.

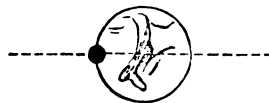
5. The chief difference, however, between the *Juno* and *Mars* series consists in the use of the reversing prism, described in Chapter II. Upon the use of this little accessory I consider the freedom of any such series of observations from systematic error in great part depends. In other words, were the apparent accuracy of the observations ever so great, and the determination of the corrections of all other kinds ever so complete, I should still look with the gravest doubt upon the result of the observations, if it were necessary to assume that there are no personal differences in measures of distance made at angles differently inclined to the vertical axis of the observer's eye. The well-known errors of double-star observers in this respect seem to render further argument on the point unnecessary. But, by simply rotating such a prism, all the observations can be made precisely as if the line joining all stars, or the planet and stars, had always a constant relation to the vertical axis of the observer's eye. This was always kept in mind, and all the measures were made with the prism so turned as to make the line joining the two objects under observation apparently horizontal.

6. In observing *Mars* another precaution was also adopted. One method

of measurement (afterwards described as Method A) was to place the star in the estimated centre of the disk of *Mars*. It was of course possible that the observer might have a bias or a tendency to place the planet too much to right or left of the true centre. But by simply rotating the prism 90° in alternate observations, this tendency is completely eliminated. In every group of four bisections (which constitutes one measure and affords one equation), two of the observations were made in one position of the prism and two with the prism rotated 90° from the first position.

7. In the other method of observation (afterwards called Method B) the image of the star is first placed in the centre of that of the planet and then immediately made to bisect one limb by motion of the segments in distance only; thus

FIG. 2.



so that the border of the planet's limb appears to bisect the disk of the star. In the next observation the image of the star, after being again centred on the disk, is made to bisect the opposite limb of the planet; thus

FIG. 3.



The segments are then reversed and the operation is repeated. When the star is very faint I think there is a tendency, in this mode of observing, to place it more outside the limb than in the case of a bright star. The effect of such a tendency, at least if both limbs are equally bright, is simply to increase the apparent diameter of *Mars*, but, as the diameter is eliminated in each group, this would have no influence on the resulting distance of centres. It would be otherwise if one limb were fainter than the other, and I believe, when there is considerable phase, that some error is caused in this way, the full

limb being always the brighter. On this account I do not think that reliance can be put on the absolute place of the planet when deduced from a faint star on one side and a bright star on the other, though the result for parallax will be perfectly reliable, because the systematic error will be the same for the evening and morning observations of the same day.

8. To make it possible to observe stars on the disk of *Mars*, and to permit of faint stars being seen on the limbs, it was necessary to reduce the brilliancy of the planet by means of a screen of wire gauze laid over the segment of the object glass through which *Mars* is viewed. The mode of action of this screen is described in the *Dun Echt Publications*, vol. ii. p. 7; but in case of the observations of *Mars* a screen of three folds of wire gauze was employed—the plan of reducing the brilliancy of the planet or star by means of an inclined screen as mentioned in the description just referred to, was found not to work well. With the triple screen the planet appeared as a sharp well-defined disk, upon which, as a background, 7th-magnitude stars could be well observed. To observe fainter stars than this upon the disk of the planet required very exceptional atmospheric conditions. Stars of 7·5 magnitude, however, could be comfortably observed on the limbs of the planet, but stars of 8th magnitude presented greater difficulty. In the case of measures of the distance of *Mars* from the star λ ,* I have still a distinct recollection of the difficulty of the observations and the great strain on the eyesight which the making of them involved.

9. In every observation means were taken to eliminate errors which may arise from change in the mean scale value. The methods employed for this purpose are best described in connection with the discussion of the observations.

* This star is estimated 8·0 mag. at Melbourne, but Prof. LEWIS BOSS sends the following estimates:—

1878.	Mag.	1878.	Mag.	} Mean 8·6;
Nov. 26	8·5	Dec. 8	8·4	
30	9·0	14	9·0	
Dec. 5	8·4	19	8·3	
6	8·7	...		

and this agrees with my own estimates, as it certainly was fainter than the stars β and μ , and was the faintest star compared with *Mars*. It was spoken of at Mars Bay as 'that wretch λ .' *lambda*.'

The Micrometer Screw.

The micrometer by which the heliometer scales are subdivided is described in vol. ii. of the *Dun Echt Publications* (pp. 1-10), and the reader is referred to that work for an account of its details.

The peculiar feature of the form of micrometer in question is, that the web frame does not move in a slide, but is mounted entirely on the screw; a point on the web does not therefore necessarily move in a straight line when the head of the screw is turned, unless the screw and its pivots are perfect. In the description of the micrometer, above referred to, it is shown that, if the screw is bent, or, if either of its two pivots are not absolutely cylindrical, part of the effect of rotating the screw will be to produce 'a couple' changing the angle between the webs and the axis of the screw. It is also shown that the errors having origin in either of the above-mentioned causes will vary proportionally with the distance of the observed point of the web from the axis of the screw. It is therefore probable that the screw errors will not be quite identical for the two scales, as the portion of the web used in reading Scale A is considerably nearer the screw than the portion used in reading Scale B. In the former investigation of the screw I had only satisfied myself that the errors of the kind in question were exceedingly small, and the observations were so arranged that a mean value of the errors was determined from the two scales.

In the present investigation, however, I determined to make these errors a matter of special research, and accordingly carried out two entirely independent determinations of the errors of the screw, in one of which all the observations were made on Scale A, in the other all on Scale B.

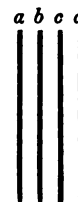
Much trouble was also taken to determine accurately not only the periodic errors of the screw, but also those smaller errors which do not recur periodically with each revolution (in general chiefly the result of wear)*—which in so short a space as *four* revolutions must necessarily be small, yet may in some cases introduce a systematic error if not accurately determined

* In the description of the Micrometer (*Dun Echt Publications*, vol. ii. p. 54), the other causes of these errors are described.

and allowed for. With regard to errors caused by wear, it is important to remark that the observations for screw-error were carried out simultaneously with the other work of the Expedition, so that the conditions of the screw, as regards wear, are identical with those in which the observations were made.

The micrometer was provided with four parallel webs, placed about 0.4 revolutions apart.

A division of the scale being placed symmetrically between the webs *c* and *d* the micrometer head is read off, then the head turned till the same division appears symmetrically between the webs *b* and *c*, and in this way is obtained a measure of the distance between the middle point between *b* and *c* and the middle point between *c* and *d*, in terms of the part of the screw employed. Thus the same space (about 0.4 revolutions) can be measured by different parts of the screw.



Similarly using the centre of the spaces *cd* and *ab* a space about 0.8 revolutions can be measured.

To measure a space of 1.2 revolutions it was necessary to find a well-defined speck on the scale and bisect it with the wires *d* and *a*.

A space of 2.0 revolutions was easily obtained by placing any division of the scale symmetrically between *b* and *c*, reading off, and then turning the micrometer screw till the next adjacent division was bisected.

Thus four series of observations for the investigation of the errors of the screw were made.

Series I. A space = 0.4 was measured beginning at each tenth division of the head from 0.0 to 3.9 (4 sets of 4 measures each).

Series II. A space = 2.0 was measured beginning at each twentieth division from 5.6 to 2.4 (4 sets of 4 measures each).

Series III. A space = 0.8 was measured beginning at each twentieth division from 5.6 to 3.4 (4 sets of 4 measures each).

Series IV. A space = 1.2 was measured beginning at each twentieth division from 1.6 to 3.4 (2 sets of 4 measures each).

These series were measured and discussed separately for each scale.

The whole of the results of the observations of the eight different series are given in the O.M., to which the reader is referred for the details of their discussion.

All the observations can be represented within quantities smaller than 0.001 by the curves of error shown on the plate, and separately for the observations made on the two scales.

The curves represent the following analytical expression :

Periodic Errors (unity = 1 p. of the Micrometer = $\frac{1}{100}$ revolution).

$$\begin{aligned} \text{On scale A. } & \cdot 152 \cos u - \cdot 059 \sin u - \cdot 011 \cos 2u + \cdot 019 \sin 2u \\ \text{,, B. } & \cdot 207 \cos u - \cdot 091 \sin u + \cdot 005 \cos 2u + \cdot 027 \sin 2u. \end{aligned}$$

Errors depending on the number of revolutions reckoned from 2^r 0.

$$\begin{aligned} \text{On scale A. } & -\cdot 0120 n + \cdot 0067 n^2 + \cdot 0045 n^3 - \cdot 0049 n^4 \\ \text{,, B. } & -\cdot 0238 n - \cdot 0071 n^2 + \cdot 0075 n^3 + \cdot 0106 n^4. \end{aligned}$$

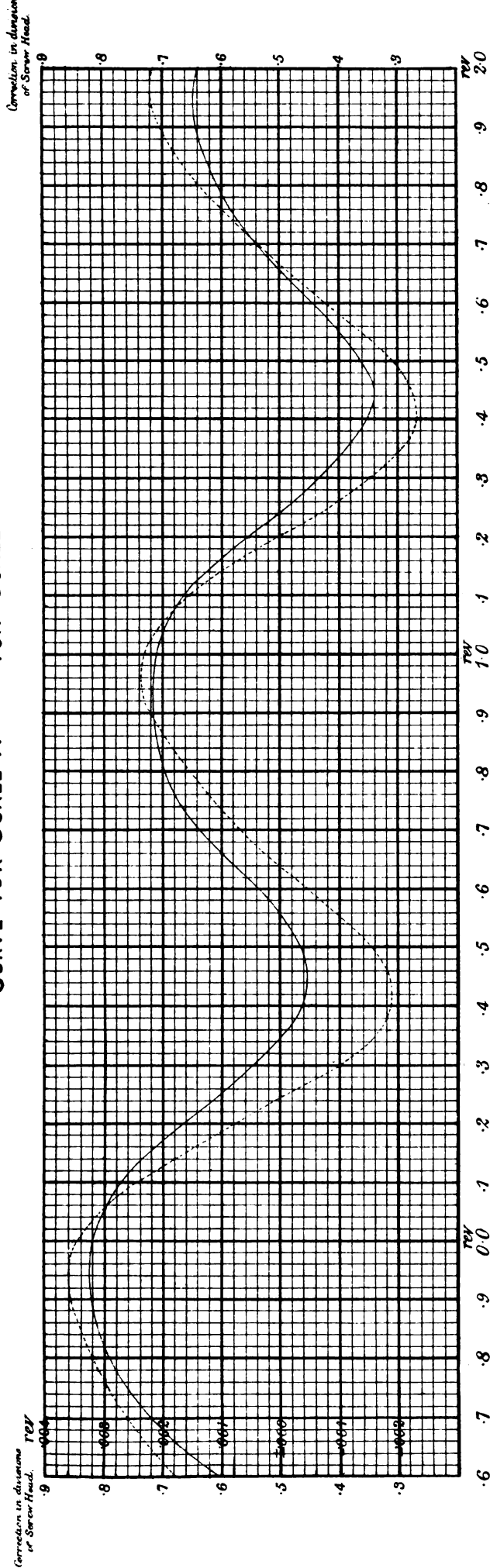
The most crucial test of the accuracy of the investigations is obtained by an inspection of the general agreement of the curves. I have already shown that if such errors as the above exist, produced by the drunkenness of the screw, by errors of its pivots, or the results of wear, their effect (from the construction of the micrometer) should be greater for Scale B than for Scale A. Notwithstanding the minuteness of the total errors, it is exceedingly satisfactory to see even these delicate points so beautifully brought out.

Since the extreme difference from the mean curve only amounts to 0.017, that curve may be safely adopted for all practical purposes, and accordingly it has been employed in the reductions for readings on both scales.

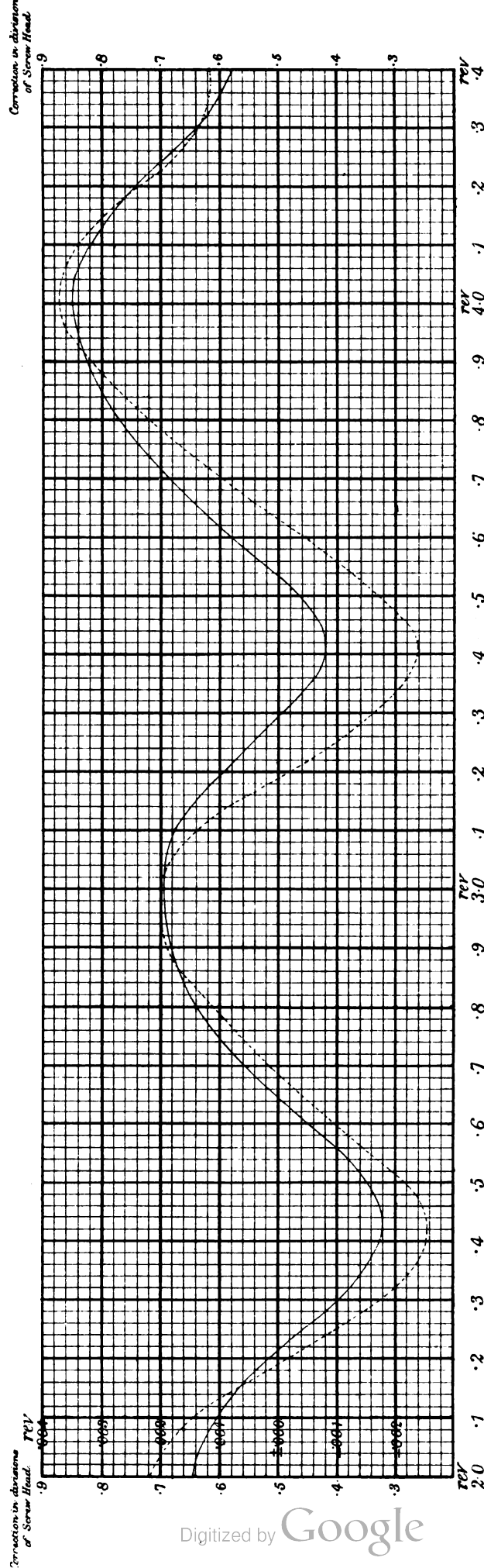
The Errors of Run.

As already stated, only one division on each scale (that following the zero* of the microscope) was read in the observations of *Mars*. It was therefore important to ascertain whether the value of the run remained constant, or varied with the temperature or other causes; and, as the observations themselves do not contain the means of eliminating this error, it was important to determine it with some precision. Previous experience had shown that the run did remain very constant, as shown by the runs deduced in the investigation of the screw, from the measures of a space = 2 rev^{ns} + run.

* The reader will understand by zero, the approximate reading of the microscope when the index or pointer is bisected,—equivalent to microscope reading 2^r 00



CURVE SHOWING THE CORRECTIONS OF THE MICROMETER SCREW AS SEPARATELY
DETERMINED FOR READINGS ON SCALE A AND SCALE B.



Scale A.		Scale B.	
1877, Sept. 27 ...	^r —'0020	1877, Oct. 20 ...	^r —'0021
Oct. 23 ...	—'0030	25 ...	—'0018
27 ...	—'0025	Nov. 17 ...	—'0012
Nov. 1 ...	—'0025	19 ...	—'0018

All these observations were made by day, and, in the very uniform temperature experienced, the run is undoubtedly very constant.

To test the effect of temperature the following observations were made over a 2-division interval (equal approximately to 4 revolutions).

	Obs.	Scale A.	Scale B.	Mean.	Temperature.
1877, Dec. 16	16	^r —'0010	^r —'0004	—'0007	F. 83'9
17	16	—'0003	—'0001	—'0002	74'5
19	16	—'0022	—'0026	—'0024	84'0
19	16	—'0019	—'0012	—'0016	74'3
21	20	—'0024	—'0015	—'0020	84'5

The above results are corrected for screw and division error, and show the correction for run in 1 revolution.

The observations in the higher temperature were made by day, and in the lower temperature by night. It is evident from inspection that the temperature coefficient must be so small as to be practically insensible in a range of temperature of 10° F., and it will undoubtedly be quite insensible in the range of ± 2° or 3° F. encountered in the observations of *Mars*.

All the observations of stars and of planet-diameters, &c., made during the expedition, in which two divisions were always read on each scale, afford a large amount of data from which the runs can be determined. A discussion of the whole gives

$$-\overset{r}{0\cdot0016} = \text{correction for run for 1 revolution of the screw.}$$

Heliumeter Observations at Ascension, July 31, 1877.

[illegible]

Formation of Instrumental Distances.

1. *When one division in each scale is read.*—In order to assist the reader who may desire to refer to the original observations (contained in the five volumes deposited with the Society), I have thought it well to give a specimen page of the original observing form, with an example of both method A and method B (*see opposite page*), and to trace the reduction of these examples from the beginning to the formation of the final equation.

The reader will observe that the notation of these micrometer readings is different from that in the O.M. Chapters VIII. and XII. In observing, I have been in the habit of reckoning the micrometer readings from the pointer; that is to say, in the original records of the observations (like the preceding examples) a micrometer reading of 0·000 is equivalent to 2·000 as entered in the observations which are recorded in the O.M. in Chapters VIII. and XII.

As a first step in the reductions I find it convenient to enter in the following computing form the readings of the scale multiplied by 2 (that is, converted into approximate screw revolutions), and the original micrometer readings increased by 2·.

The table of corrections for division-error (Table I. appended to this chapter) is prepared with the argument (Scale Reading $\times 2$); and the table for corrections of screw error (Table II.) is prepared with the argument, "Original Screw Reading $+ 2$ ·."

The index correction (or the amount to be added to Scale B—Scale A in order to make Scale B—Scale A a true measure of distance) was found to be fairly constant. A correction of $+1$ ·150 was found to be sufficiently exact for the approximate purposes required. All the observations of *Mars* have been reduced with this index error; it appears to vary to the extent of ± 2 " in course of time; but this is of no consequence, as only an approximation is required to check each individual measure, for both the effect of index error and diameter are eliminated from the mean of 4 bisections which constitute a group.

The computation is as follows:—

XLIII.

Mars and <i>n</i>	171		172		173		174	
	A.	B.	A.	B.	A.	B.	A.	B.
Scale	232'	62'	64'	232'	64'	232'	232'	62'
Following Division $\times 2$	+ '047	+ '028	+ '040	+ '031	+ '040	+ '021	+ '047	+ '028
Division Error (From Table I.)	+ 3'920	+ 3'307	+ 2'725	+ 3'095	+ 2'700	+ 3'088	+ 3'964	+ 3'245
Screw Reading ...	+ '005	+ '002	+ '005	+ '005	+ '004	+ '005	+ '005	+ '003
Screw Error and Run (From Table II.)		+ 1'150		+ 1'150		+ 1'150		+ 1'150
Index Correction ...								
Sum	235'972	66'487	66'770	236'281	66'744	236'274	236'016	66'427
Scale B—Scale A ...	- 169'485		+ 169'511		+ 169'530		- 169'589	

CCCCXX.

Mars and <i>g</i>	1384		1385		1386		1387	
	A.	B.	A.	B.	A.	B.	A.	B.
Scale	44'	250'	46'	248'	250'	44'	250'	44'
Following Division $\times 2$	+ '041	+ '038	+ '050	+ '027	+ '066	+ '027	+ '066	+ '027
Division Error (From Table I.)	+ 4'032	+ 2'310	+ 2'464	+ 3'883	+ 2'650	+ 2'919	+ 3'164	+ 2'431
Screw Readings ...	+ '005	+ '003	+ '002	+ '005	+ '004	+ '005	+ '004	+ '002
Screw Error and Run (From Table II.)				+ 1'150		+ 1'150		+ 1'150
Index Correction ...								
Sum	58'078	253'501	48'516	253'065	252'720	48'101	253'234	47'610
Scale B—Scale A ...	+ 205'423		+ 204'549		- 204'619		- 205'624	

* Original 1 revolution in error.

Here the left-hand column gives the readings of the micrometer head when the division *preceding* the zero of the micrometer is read upon. The middle column gives the reading of the scale division immediately *following* the zero of the microscope, and the third column gives the readings of the micrometer head when the latter division is bisected. The actual readings thus are

On division preceding		On division following	
Scale.	Screw.	Scale.	Screw.
143	1·843	142	3·842.

To reduce this observation with the tables formerly used, we should first have to multiply 143 and 142 each by 2, and apply separately the corrections for division error to convert into true screw revolutions; then we should apply the combined correction for screw error and run to each of the screw readings 1·843 and 3·842, and then take the mean of the whole as the reading for Scale A.

It is much simpler, however, to proceed as follows:—

- (1) Enter, from the original observations, the scale readings as they stand, multiplied by 2.
- (2) Enter the whole number of screw-revolutions increased by two, and the mean of the decimals of the screw-revolutions read on the divisions preceding and following the zero. (As these differ very slightly, they are easily entered at sight.)
- (3) Employing Table III. (given at the end of this chapter), we enter from it (with argument, “*Scale-Reading on following division multiplied by 2*”) the mean of the corrections due to division error for Scale A 143 and 142 (which mean is given by the table).
- (4) Employing Table IV. (also given at end of this chapter) we enter from it (with argument “*Screw-Reading of following division + 2*”)* the mean correction for the two screw-readings (which mean is given by the table).

The computation of the instrumental distances for group 3 is then very simply made as follows:—

* That is the unaltered screw reading on the following division as given in the O.M. Chap. VIII.

III.
Computation of Corrected Scale and Screw Readings.

Rotation Number. <i>e-g</i>	9		10		11		12	
	A.	B.	A.	B.	A.	B.	A.	B.
Scale	284'	10'	12'	284'	12'	284'	284'	10'
Following Division $\times 2$...								
Division Error ...	+ '0969	+ '0034	+ '0110	+ '0541	+ '0110	+ '0541	+ '0969	+ '0054
(From Table III.)								
Screw Readings ...	+ 3'8425	+ 3'1310	+ 2'7105	+ 3'3025	+ 2'6575	+ 3'3175	+ 3'9040	+ 3'0795
Screw Error and Run ...	+ '0058	+ '0061	+ '0063	+ '0035	+ '0058	+ '0033	+ '0060	+ '0066
(From Table IV.)								
Index Correction ...		+ 1'1100		+ 1'1100		+ 1'1100		+ 1'1100
Sum	287'9452	14'2535	14'7278	288'4701	14'6743	288'4849	288'0069	14'2015
Scale B—Scale A ...	- 273'6927		+ 273'7423		+ 273'8106		- 273'8054	

In the star computations a constant index error was not used throughout, but the foot-notes in the O.M. show what index error was used for each observation.

The whole of these computations were made and examined independently in duplicate, and afterwards compared and checked.

TABLE I.

Table of the Correction for Division Error, to be applied when one division only is read on each scale.

Argument—(Following Scale Reading) $\times 2$.

Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.
0	^r + '000	^r + '000	80	^r + '049	^r + '025	160	^r + '055	^r + '032	240	^r + '053	^r + '035
2	'000	'000	82	'044	'023	162	'055	'035	242	'049	'041
4	'000	'000	84	'042	'020	164	'052	'035	244	'053	'036
6	'007	'002	86	'040	'023	166	'053	'031	246	'058	'040
8	'007	'001	88	'038	'027	168	'054	'025	248	'060	'027
10	'007	'002	90	'044	'024	170	'053	'031	250	'066	'039
12	'010	'009	92	'042	'031	172	'052	'026	252	'068	'038
14	'012	'003	94	'041	'029	174	'050	'030	254	'071	'037
16	'016	'007	96	'042	'028	176	'045	'029	256	'070	'041
18	'008	'007	98	'041	'032	178	'046	'024	258	'076	'035
20	'012	'008	100	'048	'024	180	'044	'034	260	'076	'033
22	'012	'009	102	'050	'025	182	'053	'032	262	'082	'030
24	'013	'012	104	'049	'021	184	'049	'026	264	'077	'043
26	'023	'012	106	'054	'024	186	'046	'029	266	'086	'044
28	'023	'013	108	'051	'024	188	'045	'024	268	'084	'039
30	'025	'015	110	'056	'025	190	'046	'027	270	'088	'043
32	'026	'016	112	'056	'027	192	'046	'024	272	'087	'050
34	'035	'023	114	'054	'028	194	'047	'032	274	'090	'049
36	'024	'022	116	'059	'019	196	'048	'024	276	'094	'041
38	'025	'020	118	'053	'030	198	'050	'021	278	'100	'046
40	'035	'027	120	'063	'031	200	'051	'025	280	'095	'050
42	'039	'029	122	'053	'024	202	'044	'024	282	'098	'056
44	'041	'027	124	'057	'028	204	'054	'021	284	'096	'052
46	'050	'029	126	'055	'031	206	'040	'022	286	'098	'056
48	'043	'024	128	'052	'027	208	'050	'026	288	'097	'047
50	'041	'020	130	'056	'028	210	'051	'025	290	'103	'058
52	'040	'027	132	'055	'030	212	'049	'031	292	'090	'054
54	'041	'021	134	'057	'024	214	'047	'024	294	'092	'055
56	'038	'024	136	'060	'031	216	'042	'029	296	'094	'056
58	'038	'019	138	'064	'032	218	'041	'034	298	'096	'057
60	'038	'029	140	'065	'026	220	'048	'030	300	'098	'058
62	'048	'028	142	'060	'026	222	'039	'028			
64	'040	'035	144	'055	'029	224	'042	'029			
66	'040	'026	146	'049	'025	226	'041	'031			
68	'043	'027	148	'049	'029	228	'041	'023			
70	'043	'029	150	'050	'026	230	'043	'025			
72	'042	'024	152	'048	'029	232	'047	'031			
74	'041	'024	154	'054	'026	234	'046	'034			
76	'041	'023	156	'052	'024	236	'047	'035			
78	'041	'021	158	'054	'032	238	'048	'028			

TABLE II.

Table of the Correction for Screw Error, to be applied when one division only is read on each scale.

Argument—(Microscope Reading).

Argument.	Correction.	Argument.	Correction.
$\frac{r}{1.6}$	$+\frac{r}{.005}$	$\frac{r}{3.0}$	$+\frac{r}{.005}$
1.7	6	3.1	5
1.8	7	3.2	3
1.9	7	3.3	2
2.0	7	3.4	1
2.1	6	3.5	2
2.2	5	3.6	3
2.3	3	3.7	4
2.4	2	3.8	.5
2.5	2	3.9	5
2.6	3	4.0	5
2.7	4	4.1	4
2.8	5	4.2	3
2.9	5	4.3	2

TABLE III.

Table of the Correction for Division Error, to be applied when two divisions are read on each scale.

Argument—(Following Scale Reading) $\times 2$.

Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.	Arg.	Scale A.	Scale B.
0	+ '0000	+ '0000	80	+ '0465	+ '0245	160	+ '0550	+ '0339	240	+ '0510	+ '0384
2	'0000	'0000	82	'0429	'0218	162	'0537	'0351	242	'0513	'0389
4	'0022	'0010	84	'0412	'0213	164	'0527	'0329	244	'0558	'0401
6	'0056	'0013	86	'0393	'0250	166	'0537	'0279	246	'0591	'0355
8	'0071	'0011	88	'0411	'0255	168	'0533	'0277	248	'0631	'0324
10	'0089	'0054	90	'0431	'0275	170	'0521	'0284	250	'0671	'0377
12	'0110	'0058	92	'0417	'0301	172	'0507	'0282	252	'0695	'0374
14	'0137	'0047	94	'0414	'0284	174	'0471	'0300	254	'0703	'0390
16	'0121	'0071	96	'0411	'0302	176	'0453	'0269	256	'0728	'0378
18	'0101	'0078	98	'0441	'0282	178	'0452	'0291	258	'0759	'0340
20	'0117	'0087	100	'0485	'0248	180	'0485	'0330	260	'0791	'0318
22	'0125	'0104	102	'0495	'0234	182	'0508	'0389	262	'0798	'0368
24	'0183	'0120	104	'0519	'0227	184	'0474	'0272	264	'0815	'0435
26	'0230	'0124	106	'0528	'0242	186	'0455	'0266	266	'0850	'0414
28	'0239	'0139	108	'0537	'0244	188	'0456	'0259	268	'0861	'0411
30	'0257	'0169	110	'0560	'0259	190	'0458	'0256	270	'0875	'0467
32	'0305	'0195	112	'0549	'0277	192	'0464	'0280	272	'0884	'0496
34	'0294	'0223	114	'0566	'0236	194	'0474	'0282	274	'0917	'0451
36	'0247	'0212	116	'0564	'0244	196	'0490	'0224	276	'0968	'0438
38	'0302	'0235	118	'0581	'0305	198	'0510	'0230	278	'0974	'0483
40	'0370	'0279	120	'0581	'0274	200	'0476	'0246	280	'0963	'0532
42	'0398	'0282	122	'0551	'0256	202	'0488	'0243	282	'0970	'0541
44	'0452	'0282	124	'0559	'0295	204	'0469	'0215	284	'0969	'0541
46	'0461	'0265	126	'0534	'0290	206	'0443	'0241	286	'0972	'0517
48	'0418	'0221	128	'0540	'0275	208	'0498	'0255	288	'0997	'0523
50	'0402	'0235	130	'0558	'0293	210	'0498	'0280	290	'0964	'0556
52	'0401	'0237	132	'0560	'0271	212	'0478	'0278	292	'0971	'0563
54	'0392	'0222	134	'0582	'0275	214	'0444	'0265	294	'0980	'0570
56	'0379	'0213	136	'0617	'0315	216	'0415	'0313	296	'0980	'0570
58	'0380	'0242	138	'0641	'0290	218	'0446	'0319	298	'0980	'0570
60	'0429	'0286	140	'0621	'0259	220	'0434	'0292	300	'0980	'0570
62	'0440	'0312	142	'0572	'0272	222	'0402	'0286			
64	'0401	'0303	144	'0520	'0269	224	'0417	'0300			
66	'0414	'0265	146	'0494	'0271	226	'0410	'0272			
68	'0430	'0278	148	'0496	'0275	228	'0416	'0243			
70	'0425	'0266	150	'0488	'0273	230	'0451	'0283			
72	'0413	'0240	152	'0508	'0272	232	'0470	'0328			
74	'0404	'0233	154	'0529	'0246	234	'0468	'0346			
76	'0404	'0222	156	'0532	'0280	236	'0475	'0314			
78	'0449	'0235	158	'0546	'0324	238	'0503	'0317			

TABLE IV.

Table of the Corrections for Screw Error, to be applied when two divisions are read on each scale.

Mean of Screw Reading on Following Division.	·00	·02	·04	·06	·08
^r 1·8	^r + ·0090	^r + ·0091	^r + ·0091	^r + ·0092	^r + ·0093
1·9	93	93	93	93	93
2·0	92	91	90	88	86
2·1	83	80	77	75	72
2·2	69	66	63	59	55
2·3	53	50	48	46	45
2·4	44	43	42	42	43
2·5	44	45	47	49	50
2·6	52	54	56	58	60
2·7	62	63	65	66	67
2·8	68	69	70	71	72
2·9	72	72	72	72	72
3·0	71	70	69	68	66
3·1	64	62	60	57	54
3·2	51	49	44	40	38
3·3	35	33	31	30	29
3·4	28	28	28	27	28
3·5	29	30	32	34	35
3·6	37	39	41	44	46
3·7	48	50	51	52	54
3·8	55	57	58	60	59
3·9	60	60	61	61	60
4·0	60	60	59	58	57
4·1	55	53	51	49	46
4·2	44				

CHAPTER VII.

THE REFRACTION AND METEOROLOGICAL OBSERVATIONS.

The refraction is one of the most important corrections which have to be discussed in observations for planetary parallax. It affects the apparent position of the object in the same sense as parallax, and, without special precautions, it is apt to become a source of systematic error. Fortunately these precautions can be made very effectual, and they are also easy of application.

- (1) Since the parallax varies as the sine, and the refraction as the tangent of the zenith distance, very little advantage in parallax factor is gained by making the observations at extreme zenith distances, whilst greatly increased refraction is encountered. Therefore refined observations for parallax should not be made at distances much exceeding 70° , and the law of refraction is known with tolerable certainty to that zenith distance. This precaution has been followed.
- (2) Frequent readings of the Barometer and Thermometer, for the correction of the mean refraction, were made every night.
- (3) Stars above, as well as stars below, the planet were always observed, so that the effect of an abnormal increase of refraction would be to increase the apparent distance between the stars of comparison, and to increase the apparent distance of the planet from each star, nearly proportionally with the distances. Such increase would produce the same effect on the distance as an increase of the scale value, and this is entirely eliminated in the method of reduction employed. A term depending on the refraction, however, has been carried throughout the investigation for parallax.

The refractions have been computed by the following very rigid formulæ which are given by Dr. HUGO SEELIGER in his *Theorie des Heliometers*, p. 96.

$$\begin{aligned}\Delta s &= as [1 + \tan^2 \zeta_0 \cos^2 (p_0 - q_0)] + a's \sin^2 (p_0 - q_0) \\ \Delta p_0 &= -a [\tan^2 \zeta_0 \sin (p_0 - q_0) \cos (p_0 - q_0) + \sin q_0 \tan \zeta_0 \tan \delta_0] \\ &\quad - a' [\tan \zeta_0 \sin q_0 \tan \delta_0 - \sin (p_0 - q_0) \cos (p_0 - q_0)].\end{aligned}$$

where

- s and p_0 = the *apparent* distance and mean position angle respectively.
- ζ_0 = the *true* zenith distance of the middle point between the objects.
- q_0 = the *parallactic* angle of the middle point between the objects.
- δ_0 = the *declination* of the middle point between the objects.

and

$$a = a \times \beta^A \times \gamma^A.$$

where a and a' are taken from the tables given in the work in question, and where β^A and γ^A are the usual corrections depending on the readings of the meteorological instruments.

In the O.M. I have given the details of the methods which I have adopted to facilitate and check the computations of the corrections for refraction, together with the auxiliary tables I have employed for this purpose. I have also given the separate readings of the meteorological instruments made during the observations. I have not considered it necessary to print these in the present memoir.

CHAPTER VIII.

HELIOMETRIC TRIANGULATION OF COMPARISON STARS.

In the O.M. this chapter contains the details of all the observations made in connection with the Heliometric triangulation of the stars of comparison.

It has not been thought necessary to reprint these observations in detail, and the reader is referred to the O.M.

CHAPTER IX.

COMPARISON OF THE RESULTS OF HELIOMETER AND MERIDIAN OBSERVATIONS
OF COMPARISON STARS.

It was my intention, from the beginning, to supplement the meridian observations of the comparison stars with a heliometric triangulation—that is, to measure all the distances of the stars from each other, so far as the range of the instrument ($2^{\circ} 7'$) would permit.

In former papers I have shown that although the Heliometer is capable of extreme accuracy in the measurement of proportional distances, it cannot, without special precautions, be employed, with its highest efficiency, in the measurement of absolute distances. It is, in fact, impossible to determine with great accuracy* the scale-value of a Heliometer at any particular instant, for the simple reason that it is impossible to determine exactly all the conditions which affect it.

Given a normal condition of the observer's eye, a normal focal adjustment, a uniform temperature of all parts of the instrument, and that temperature accurately known, it would then be possible, if this condition of things could be repeated again and again, to determine the scale value for these conditions with a high accuracy. But it much more generally happens that the combinations of all these circumstances are different. It is seldom that all parts of the instrument are exactly of the same temperature; and the computation of the effects of different temperatures of the tube, of the scale and of the object glass, upon the scale value, is always affected with considerable uncertainty, due partly to the difficulty of determining these temperatures and the corresponding coefficients, but chiefly to differences in these coefficients arising from the variation of the adaptive state of the observer's eye.

* I refer of course to such accuracy as may be obtained in the mean of a number of measures; for example, to make certain within $\pm 0''.05$ in an arc of 2° that a particular error is due simply to error of bisection and not to systematic error of scale value.

When large angular distances have to be measured accurately, it thus becomes necessary to have some invariable standard distance, the measurement of which, in terms of the scale or screw, gives a direct and simple means of obtaining the scale value and its variation.

I have employed, for such a standard, the distance between the stars *e* and *g*. A reference to the Plate (which gives a graphic representation of the measures made in the triangulation) will show that these standard stars *e* and *g*, lie nearly in the centre of the group of stars compared with *Mars*, and the distance between them is nearly the greatest that the Heliometer can measure.

The programme of observation was

- (1) Make a complete measure of the standard distance *e-g*.
- (2) Make a complete measure of each of three distances in the triangulation.
- (3) Repeat a complete measure of the standard distance *e-g*.

Thus the standard distance *e-g* was assumed to be for 1877.0 =

$$*7054''.796 + \Delta S \quad (1).$$

Each actual measurement of the distance *e-g* was converted into arc by the formula

$$\Sigma \times 25''.727 + nZ \quad (2).$$

where

Σ is = the observed distance expressed in revolutions of the Micrometer Screw, corrected for refraction, aberration, and proper motion to 1877.0.

n = $\frac{\text{the observed distance expressed in seconds of arc}}{10,000}$.

Z = the error which would be produced in an arc of 10,000'' by the erroneous scale value employed.

Equating the expressions (1) and (2) we get

$$Z = \frac{7054.796 - \Sigma \times 25.727}{n} + \frac{\Delta S}{n} \quad (3).$$

* Computed from the co-ordinates of the preliminary catalogue (*Monthly Notices*, Dec. 1878).

The standard distance ($e-g$) was measured in all 58 times on 30 different nights. Each of these measures gives a determination of Z in terms of ΔS , thus the value of Z is determined at 58 different epochs.

Take, for example, the observations of July 22,* the work of the first evening in Garrison.

Stars e-g.

Sidereal Time.	Rotation No. of Group.	α .	$\alpha \times 25''.727$.	Deduced Value of S by Eq. (3).
h m 18 31	3	274°2771	7056''33	$-2''.17 + 1'.42\Delta S$
20 4	7	2462	55'53	$-1'.04 + 1'.42$
21 45	11	2299	55'11	$-0'.44 + 1'.42$

* It is very seldom that the value of Z amounts to so large a quantity as $2''.17$. I have simply quoted the first night's observations for illustration, that I may not be open to the suspicion of having selected unusually favourable conditions, and it happens that the results of July 22 are exceptionally discordant. Take, for example, the following, selected at random from the cases in which the standard stars have been observed three times on the same night:—

July 24.		November 29.		November 30.	
Sidereal Time.	Z .	Sidereal Time.	Z .	Sidereal Time.	Z .
h m 18 34	$-1''.24$	h m 0 43	$+0'.61$	h m 1 37	$0''.00$
20 10	$-0'.71$	1 52	$-0'.33$	3 14	$-0'.27$
23 42	$-0'.68$	3 29	$-0'.24$	4 15	$+0'.10$

where, to each value of Z , the constant $1'.42\Delta S$ must, of course, be added.

I have examined at great length the possible causes of these variations of scale value. Most certainly they cannot be traced to any errors of refraction, but beyond this I am unable to speak definitely. There seemed to be some connection between the zenith distance and the scale value, on the mean of a large number of observations east of the meridian, which, at one time I thought might be due to some change of focus in the eye of the observer, produced by the different inclination of the axis of the observer's eye; but this hypothesis received no confirmation, from the observations made west of the meridian.

I came finally to the conclusion that such changes of scale value might probably be due to the difference of rate of cooling of the object glass from that of other parts of the instrument, for the observations east of the meridian were generally begun as soon after sunset as

On the same evening 6 other pairs of stars were observed, for which the values of Z were graphically interpolated as follows :—

Rotation Number of Group.	Stars Observed.	Sidereal Time.	Z .
		h m	
4	$a-b$	18 53	$-1^{\circ}82 + 1.42\Delta S$
5	$c-b$	19 16	$-1^{\circ}59 + 1.42$
6	$c-\mu$	19 42	$-1^{\circ}25 + 1.42$
8	$d-\mu$	20 36	$-0^{\circ}75 + 1.42$
9	$e-\mu$	20 58	$-0^{\circ}61 + 1.42$
10	$f-g$	21 23	$-0^{\circ}50 + 1.42$

The observations of these stars furnish the following results :—

	Σ .	$\Sigma \times 25''727 + nZ$.	Tabular Distance.*	$C-O$.
$a-b$	82.9873	$2134^{\circ}62 + .30\Delta S$	$2135^{\circ}12$	$+0^{\circ}50 - .30\Delta S$
$c-b$	224.2285	$5767^{\circ}81 + .82$	$5768^{\circ}23$	$+0^{\circ}42 - .82$
$c-\mu$	208.9277	$5374^{\circ}42 + .76$	$5375^{\circ}86$	$+1^{\circ}44 - .76$
$d-\mu$	248.3024	$6387^{\circ}60 + .91$	$6387^{\circ}45$	$-0^{\circ}15 - .91$
$e-\mu$	112.2325	$2887^{\circ}22 + .41$	$2887^{\circ}02$	$-0^{\circ}20 - .41$
$f-g$	124.9594	$3214^{\circ}67 + .46$	$3214^{\circ}85$	$+0^{\circ}18 - .46$

If now it could be assumed—

- (1) That the distances derived from the meridian observations are affected by no systematic error, and
- (2) That the Heliometer Scale value is uniform throughout its extent.

Then on these two assumptions we could obtain the error of the standard distance by simply putting the residuals ($C-O$) of the preceding example) = 0,

the objects were visible. I found it very difficult to determine any general law, as the results are masked by the varying condition of the observer's eye. There is therefore only one method of eliminating such errors, viz., to employ some such standard of measure as that adopted in the previous investigation; or, in the case of observing a planet, to select well-determined stars of comparison on opposite sides of it.

* Computed from the co-ordinates of the preliminary catalogue (*Monthly Notices*, Dec. 1878).

and solving the equations so formed by the method of least squares, having regard to their weights. But it is already an ascertained fact that at each Observatory the right ascensions require some correction depending on the magnitude (*Monthly Notices*, p. 112, vol. xxxix.).

Now the places of Chapter VIII., from which the tabular distances were computed, have been derived on the assumption that from the mean of the results of twelve Observatories the systematic error depending upon magnitude will disappear. It however does not follow, as a matter of certainty, that this will be so, and it has therefore formed part of my programme to derive these corrections from comparison with the heliometer measures.

Let $*_1$ and $*_2$ denote two stars of different magnitude, of which $*_1$ is the brighter.

„ s and p_0 denote the distance and position angle respectively, p_0 being reckoned in the usual way from North through East, the brighter star being taken for origin.

„ m_1 and m_2 denote their magnitudes, or rather the difference of their magnitudes from 4.5 magn. (according to this for a 6th magn. star $m = 1.5$), m_1 being the magn. of the brighter star.

If the systematic error of R.A. depending upon magnitude, could be supposed to vary directly as the magnitude, then the true right ascensions of the stars $*_1$ and $*_2$, apart from accidental errors, would be

$$\alpha_1 + m_1 a \quad \text{and} \quad \alpha_2 + m_2 a$$

respectively, where a is a factor to be determined.

But an inspection of the curves obtained by Prof. BAKHUYZEN (*Monthly Notices*, vol. xxxix. p. 122) shows that it is probably desirable to take also into account a term depending on the square of the magnitude.

Hence the corrections for magnitude will become

$$m_1 a + m_1^2 b \quad \text{and} \quad m_2 a + m_2^2 b$$

and the true difference of right ascension between the stars will be

$$\alpha_2 - \alpha_1 + (m_2 - m_1)a + (m_2^2 - m_1^2)b.$$

If the tabular distance (s) has been computed from values of a_1 and a_2 to which corrections for magnitude have not been applied, then the true tabular distance will be

$$s + \sin p_0 \cos \delta_0 (m_2 - m_1)a + \sin p_0 \cos \delta_0 (m_2^2 - m_1^2)b \quad (4).$$

Now with regard to the second question—Is the Heliometer Scale-value uniform throughout?

From the investigation of the scale-value in connection with the measures of *Juno* at Mauritius (*Dun Echt Publications*, vol. ii. p. 126) it has been shown that, within very narrow limits, the assumption would be justified. At the same time it must be noted that towards the extremity of the scale the probable error of the investigation of the division errors amounts $\pm 0''.10$ (*Dun Echt Publications*, vol. ii. p. 48), and therefore I thought it not unlikely, with the considerable material at disposal, that an expression might be found which would more closely represent the true measures of the different angles. The investigation, at least, cannot be without interest, as affording a crucial test of the accuracy of the investigation of the division errors, and the reliability of Heliometer Observations.

I shall therefore suppose that all the measured distances require a correction which may be expressed in terms of the distance and its square—that is to say, the true observed distance may be expressed by

$$25.727\Sigma + nZ + n^2d \quad (5).$$

Equating the expressions 4 and 5 we obtain

$$\sin p_0 \cos \delta_0 (m_2 - m_1)a + \sin p_0 \cos \delta_0 (m_2^2 - m_1^2)b - n^2d + s - (25.727\Sigma + nZ) = 0 \quad (6).$$

Each observation of distance, when reduced, as in the preceding example, will therefore furnish an equation of the form of Equation (6), and for each we find from the column C-O (O.M. Chap. IX. Table IV.) the equivalent for the expression $\{s - (25.727\Sigma + nZ)\}$.

Thus each observation will furnish an equation involving the four unknown quantities

$$a \quad b \quad d \quad \text{and} \quad \Delta S.$$

But all the equations will not have equal weight. Each will be affected by three sources of error which we may call ϵ_{II} , ϵ_{III} and ϵ_{IV} .

ϵ_{II} is the *mean error* of the determination of scale-value at the time of measurement. It will therefore depend upon the accuracy of the measurement of the standard distance, and the proportion of the measured distance to the standard distance. Thus if ϵ is the *mean error* of a measurement of the standard distance we have

$$\epsilon_{II} = \frac{s}{S} \epsilon.$$

ϵ_{III} is the *mean error* of the actual measurement of the observed distance, so far as accidental error of *bisection* is concerned, but apart from the systematic error of the scale-value at the time. In other words, it is the mean error of observation, if the scale value at the time of observation were accurately known. I have already shown (*Monthly Notices*, vol. xxxiv. p. 286), that, were it not for change of scale-value, all angles would be measured by this Heliumeter with equal accuracy, the goodness of the images not being sensibly different for different distances, because the slides in which the segments move are cylindrical.

Hence

$$\epsilon_{III} = \epsilon.$$

ϵ_{IV} is the *mean error* of the Tabular distance (s) computed from the data of the results of the meridian observations (*Monthly Notices*, Dec. 1878), by the formula

$$\epsilon_{IV}^2 = (\sin p_0 \cos \hat{c}_0)^2 (\epsilon_a^2 + \epsilon_b^2) + \cos^2 p_0 (\epsilon'_a{}^2 + \epsilon'_b{}^2) \quad (7).*$$

where

ϵ_a	and	ϵ_b	are the mean errors of the stars a and b in R.A. in seconds of arc.
ϵ'_a	and	ϵ'_b	" " " in Dec. "

* The separate values ϵ_{IV}^2 are given in the O.M. Chap. IX. Table III.

In order to deduce the value of ϵ I have proceeded as follows.

Nineteen of the pairs of stars have been observed twice. From the discordances of these double observations I have computed for each pair the value of

$$\frac{[vv]}{n-1} = \text{twice the square of half the difference between the two observations.}$$

The mean of these 19 values of $\frac{[vv]}{n-1}$ should fairly represent the mean value of

$$\epsilon_{II}^2 + \epsilon_{III}^2.$$

or, what is the same thing, should fairly represent the mean value of

$$\frac{s^2}{S^2} \epsilon^2 + \epsilon^2.$$

Computing for each pair of stars the value of $\frac{s^2}{S^2}$ the sum of these 19 values is found to be 13.33

The sum of the 19 values of $\frac{[vv]}{n-1}$ is 1.5556;

whence

$$13.33 \epsilon^2 + 19 \epsilon^2 = 1.5556.$$

and

$$\epsilon^2 = 0.0481.$$

or

$$\epsilon = \pm 0''.22.$$

The *mean error* then of any equation of the type of (6) is

$$\sqrt{\frac{\epsilon_{II}^2 + \epsilon_{III}^2}{n} + \epsilon_{III}^2}.$$

Where n is the number of observations on different days. Or adopting for weight *unity* a *mean error* of $\pm 0''.5$, the expression for the weight of each equation becomes

$$p = \frac{0.25}{\frac{\epsilon_{II}^2 + \epsilon_{III}^2}{n} + \epsilon_{III}^2}.$$

in which

$$\epsilon_{II}^2 = .0481 \frac{s^2}{S^2}$$

$$\epsilon_{III}^2 = .0481$$

and

ϵ_{III}^2 is computed by formula (7).

In this way the following equations have been formed from the observations. Those observations only have been omitted which involve the stars ϵ and θ . The star ϵ is a close double-star, and the personal equation of its observation by meridian observers might be very different from that of a single star whose magnitude is the same as that of the components of ϵ . The other star θ was only observed at a few of the observatories.

*Equations resulting from the Comparison of the Distances observed with the Heliometer, and the Distances computed from the results of Meridian Observations.**

Stars.		<i>p</i>
<i>a-b</i>	$0.18a + .65b - .05d - .30\Delta S + ".24 = 0$	4.63
<i>a-a</i>	$3.15 + 20.47 - .20 - .63 + .06$	2.19
<i>a-c</i>	$0.00 \quad .00 - .56 - 1.07 - .39$	2.78
<i>b-a</i>	$1.86 + 13.42 - .27 - .74 + .29$	2.19
<i>c-b</i>	$- .57 - 2.11 - .33 - .82 + .30$	3.01
<i>c-a</i>	$- .61 - 3.98 - .50 - 1.01 - .54$	1.97
<i>b-a</i>	$- 1.75 - 14.36 - .16 - .56 - .68$	2.17
$\mu-a$	$- 1.21 - 10.31 - .46 - .96 - .05$	2.55
<i>d-a</i>	$- 1.84 - 14.75 - .54 - 1.04 - 1.55$	1.64
<i>c-β</i>	$.62 + 2.89 - .49 - .99 - .52$	1.98
<i>c-μ</i>	$1.57 + 7.83 - .29 - .76 + 1.44$	1.98
<i>c-γ</i>	$2.39 + 13.13 - .44 - .94 + .42$	2.34
<i>β-μ</i>	$.14 + .91 - .14 - .53 - .04$	2.41
<i>d-β</i>	$- .14 - .86 - .18 - .60 + .60$	2.29
<i>β-γ</i>	$.47 + 3.35 - .43 - .93 + .26$	2.78
<i>e-β</i>	$- .18 - 1.10 - .22 - .67 + .11$	3.01

* The reader is referred to the O.M. Chap. IX. for full details of the data from which these equations have been formed.

Equations resulting from Comparison of the Distances, etc.—continued.

Stars.	...	p
$d-\mu$	$-.10a - .62b - .41d - .91\Delta S - .15 = 0$	1.93
$\mu-\gamma$	$.36 + 2.70 - .09 - .42 - .17$	2.19
$e-\mu$	$-.43 - 2.81 - .08 - .41 - .20$	2.27
$d-e$	$.00 \quad 0.00 - .26 - .73 - .09$	2.29
$d-\delta$	$1.47 + 11.02 - .25 - .70 + .01$	1.94
$e-\gamma$	$-.11 - .78 - .11 - .47 + .01$	3.73
$e-\delta$	$.90 + 6.73 - .35 - .84 + .70$	2.05
$f-\delta$	$-1.91 - 13.40 - .08 - .41 - .17$	2.08
$g-\delta$	$-.79 - 6.13 - .26 - .73 - .06$	2.19
$i-\delta$	$-3.36 - 17.44 - .54 - 1.05 - 1.29$	1.63
$f-g$	$.14 + .84 - .10 - .46 + .18$	2.61
ψ, f	$-.44 - 1.10 - .34 - .83 - .88$	2.03
$h-f$	$-.45 - 1.91 - .21 - .64 - .06$	2.19
$i-f$	$-1.24 - 3.96 - .29 - .77 - .12$	1.97
$f-k$	$.29 + 1.57 - .43 - .93 - .10$	1.81
$f-l$	$.49 + 2.68 - .30 - .78 + .42$	1.81
$f-\lambda$	$.98 + 5.87 - .55 - 1.05 + .17$	1.52
$h-g$	$-1.51 - 7.55 - .04 - .30 - .40$	2.43
$k-g$	$-.24 - 1.53 - .15 - .56 - .13$	2.25
$l-g$	$-.23 - 1.47 - .38 - .87 - .37$	1.94
$g-\lambda$	$.18 + 1.23 - .54 - 1.04 + .48$	1.66
ψ, i	$.57 + .40 - .11 - .48 - .11$	2.25
ψ, l	$1.92 + 5.77 - .47 - .97 + .61$	1.81
ψ, ζ	$3.41 + 11.92 - .45 - .95 + .84$	1.66
$h-k$	$0.75 + 3.53 - .04 - .29 + .03$	2.69
$h-l$	$0.71 + 3.33 - .26 - .73 - .26$	2.21
$h-\lambda$	$1.44 + 7.47 - .34 - .83 + 1.54$	1.95
$i-l$	$1.03 + 3.80 - .14 - .53 + .64$	2.53
$i-\lambda$	$1.78 + 7.49 - .30 - .78 + .52$	2.06
$i-\zeta$	$2.70 + 11.34 - .15 - .56 + .94$	2.12
$i-\eta$	$3.35 + 16.43 - .48 - .99 + .68$	1.68
$l-k$	$.00 \quad .00 - .37 - .87 + .05$	2.10
$k-\lambda$	$.28 + 1.83 - .38 - .87 + .59$	1.98

Equations resulting from Comparison of the Distances, etc.—continued.

Stars.	...	<i>p</i>
<i>l-λ</i>	$.44a + 2.89b - .04d - .30\Delta S + .20'' = 0$	2.45
<i>l-ζ</i>	$.44 + 2.88 - .22 - .66 + .66$	2.36
<i>l-η</i>	$1.11 + 8.01 - .29 - .76 + .29$	1.97
<i>m-l</i>	$- .61 - 3.23 - .51 - 1.02 - .18$	2.61
<i>ζ-λ</i>	$.00 .00 - .25 - 0.71 + .26$	2.38
<i>λ-η</i>	$.53 + 4.05 - .17 - .59 + .39$	2.25
<i>m-λ</i>	$- 1.03 - 5.98 - .26 - .72 + .04$	2.06
<i>ζ-η</i>	$.54 + 4.14 - .14 - .53 + .41$	2.29
<i>n-ζ</i>	$- 1.47 - 8.10 - .26 - .73 - .07$	1.97
<i>m-η</i>	$- .46 - 2.97 - .28 - .75 + .41$	2.34
<i>n-η</i>	$- 1.33 - 8.22 - .12 - .50 - .31$	2.53
<i>q-η</i>	$- 1.38 - 9.27 - .39 - .88 - .92$	1.91
<i>m-q</i>	$.18 + 0.87 - .17 - .59 + .28$	3.17
<i>m-r</i>	$.18 + 0.84 - .53 - 1.03 + .01$	2.58
<i>n-q</i>	$.21 + 0.96 - .48 - .99 + .06$	3.09
<i>n-r</i>	$.38 + 1.71 - .54 - 1.05 + .42$	2.66
<i>n-s</i>	$.95 + 4.73 - .42 - .92 + .51$	2.63
<i>q-r</i>	$.00 .00 - .10 - .44 + .43$	3.42
<i>r-s</i>	$.04 + .21 - .42 - .92 + .16$	3.25
<i>t-r</i>	$- .16 - .73 - .54 - 1.04 - .08$	3.05
<i>t-s</i>	$- .96 - 4.78 - .04 - .29 + .31$	3.25

Treating these 70 equations by the method of least squares, having regard to the weight of each, the following normal equations are obtained:—

$$\begin{aligned}
 216.89a + 1184.55b - 9.84d - 23.62\Delta S &= - 59.31 \\
 1184.55 + 6930.80 - 36.69 - 91.09 &= - 322.83 \\
 - 9.84 - 36.69 + 17.44 + 39.57 &= + 3.70 \\
 - 23.62 - 91.09 + 39.57 + 93.74 &= + 11.61.
 \end{aligned}$$

The solution of which gives

$$\left. \begin{aligned}
 a &= -0.2736 \text{ with weight } 13.7 \\
 b &= .00125 \quad \quad \quad \text{''} \quad 444.4 \\
 d &= -1.5776 \quad \quad \quad \text{''} \quad 0.73 \\
 \Delta S &= 0.7221 \quad \quad \quad \text{''} \quad 3.91
 \end{aligned} \right\} \text{First system.}$$

If we neglect the term d (that is, assume the scale value to be uniform for all angles), we obtain

$$\left. \begin{array}{l} a = -0.2543 \text{ with weight } 13.7 \\ b = -0.00236 \quad \text{,,} \quad 443.4 \\ \Delta S = .0575 \quad \text{,,} \quad 87.7 \end{array} \right\} \text{Second system.}$$

If we neglect the terms a and b (that is, omit the terms depending on the magnitude of the stars of comparison), we obtain

$$\left. \begin{array}{l} d = -1.6313 \text{ with weight } 0.734 \\ \Delta S = 0.8125 \quad \text{,,} \quad 3.92 \end{array} \right\} \text{Third system.}$$

If we omit the terms depending on magnitude, and assume also the uniformity of the scale value, we obtain

$$\Delta S = 0.1239 \text{ with weight } 93.74 \text{ Fourth system.}$$

When the values of the unknown quantities found by these four systems of reduction have been substituted in the original equations (after the latter have been reduced to Weight 1 by being multiplied by $\sqrt{p}$) we obtain the following :—

...	Including Corrections for Magnitude.		Without Corrections for Magnitude.	
	First System.	Second System.	Third System.	Fourth System.
Sum of the Squares of the Residuals	20.3181	22.0992	35.1152	36.8392
Square of Mean Error of 1 Observation of weight = 1	0.3079	.3298	.5164	.5339
Mean Error of 1 Observation of weight = 1	$\pm 0''.554$	$\pm 0''.574$	$\pm 0''.719$	$\pm 0''.737$

In computing the weights, an observation of Weight 1 was supposed to be affected with a mean error of $\pm 0''.50$, and we actually obtain from the first system a mean error of $\pm 0''.55$; this agreement is very satisfactory; for in computing the weights no allowance was made for outstanding accidental

errors of division of the scale, or for possible error in interpolating the value of Z . Besides this, no allowance was made for the accidental tabular errors of magnitude, and thus the small discrepancy may easily be accounted for.

The introduction of the terms a and b , depending on the magnitude, thus largely reduces the sum of the squares of the residuals; but the importance of the reduction so produced is only clearly seen when we remember that each equation is affected by three sources of error already described as ϵ_{tab} , ϵ_{div} and ϵ_{mag} , only one of which (ϵ_{mag}) is affected by the introduction of the corrections for magnitude.

I have endeavoured to estimate the real diminution of error in the distances computed from the meridian observations by the introduction of the corrections for magnitude. Allowing an average error whose square amounts to $\cdot 0200$ for outstanding errors in the investigation of the division errors of the scale and in the interpolation of the values of Z , I then find that the resulting mean values of ϵ_{mag}^2 are

Including errors due to magnitude $\epsilon_{\text{mag}}^2 = \cdot 1470$

Eliminating errors due to magnitude $\epsilon_{\text{mag}}^2 = \cdot 0600$

or, in fact, that the neglect of the errors due to magnitude increases the square of the mean error of the tabular distances nearly as $2\frac{1}{2}$ to 1.

But further. If we take the First and Second System together, and the Third and Fourth System together, we find strong evidence that the term d (depending on the square of the distance) is not a term having a real existence, or at least its existence cannot be determined with certainty from the data at our disposal. In both Systems it comes out with very small weight ($0\cdot 73$) or with a mean error in the former system of $\pm 0''\cdot 65$, and in the latter of $\pm 0''\cdot 84$, equal in the latter case to half the amount of the quantity itself. The term b also seems to have no reality. It comes out

In First System $= +\cdot 0012 \pm \cdot 0026$

In Second System $= -\cdot 0024 \pm \cdot 0026$.

Rejecting therefore terms d and b (that is, assuming the outstanding scale errors to be accidental, and that the corrections of the right ascensions for magnitude vary directly as the magnitude), we obtain from the solution of the normals in a and ΔS

$$\left. \begin{array}{l} a = -0.2673 \text{ with weight } 216.89 \\ \Delta S = 0.0565 \quad \quad \quad \quad \quad 91.16 \end{array} \right\} \text{ Fifth system.}$$

Substituting these values in the original equations reduced to $p = 1$, we find

$$\begin{array}{l} \text{Sum of the Squares of the Residuals } \dots \dots = 22.9883 \\ \text{Square of Mean Error of 1 Observation } (p = 1) = .3381 \\ \text{Mean Error of 1 Equation } (p = 1) \quad \dots \dots = \pm .581. \end{array}$$

Practically the results of all these discussions are identical, as the following comparison shows :—

Corrections for Magnitude according to the Different Systems of Reduction.

Value of m .	Corresponding Magnitude.	Corresponding Corrections.		
		First System.	Second System.	Fifth System.
1	5.5	— 0".27	— 0".26	— 0".27
2	6.5	— .54	— .52	— .54
3	7.5	— .81	— .78	— .80
4	8.5	— 1.08	— 1.06	— 1.07
5	9.5	— 1.34	— 1.33	— 1.34

Corrections to the Distances measured if computed with a uniform Screw Value of 25".727, according to the Different Systems.

For a Distance.	Corresponding Corrections.				
	First System.	Second System.	Third System.	Fourth System.	Fifth System.
1000"	+ 0".08	+ ".01	+ ".01	+ ".10	+ ".01
2000	+ .14	+ .02	+ .17	+ .04	+ .02
3000	+ .16	+ .03	+ .20	+ .05	+ .02
4000	+ .16	+ .03	+ .20	+ .07	+ .03
5000	+ .12	+ .04	+ .17	+ .09	+ .04
6000	+ .05	+ .05	+ .10	+ .11	+ .05
7000	— .05	+ .06	+ .01	+ .12	+ .06

There seems no doubt whatever, then, as to the reality of the required corrections for magnitude—and it is worthy of remark that in the First, Second, and Fifth Systems (where errors due to magnitude are taken into account)—the corrections depending on difference of scale-value for different angles may be considered insensible, as they are less than the probable error of their determination.

The tendency of the whole investigation is to confirm the accuracy of the investigations of the errors of the scales, and the reliability of such Heliometer measures.

But I must confess that the largeness of the systematic error of the meridian observations, due to magnitude, very much surprised me. I did not think it possible that, in the mean of the results of twelve of the principal Observatories in the world, it would be possible to find a systematic error of any kind amounting to a second of arc. I searched for the origin of the discordance in my own work, but I could find nothing of the kind. If, for example, the effect of interposing a wire gauze screen in front of one half of the object glass (as was done to diminish the light of the brighter star) could be to alter the apparent place of the star, the systematic error so produced must be an increase or diminution of the observed distance by an amount having some connection with the difference of magnitude of the two stars. But if we introduce into the equations a term depending on the difference of magnitude of the stars, either as affecting the distances absolutely, or in proportion to the distance, the final result will be the same, because no systematic error in the observed distances can explain why it should be that when the brighter star precedes the fainter star the tabular distance is less than the observed distance, and when the fainter star precedes the brighter star the tabular distance is greater than the observed distance.

Anxious, however, to obtain still further confirmation, I selected a list of stars of similar declination and magnitude with the *Mars* stars, and including many of them. These test stars were so selected that the mean right ascension and mean declination of the bright and faint stars was, as nearly as possible, the same; and there was sufficient interval between the right ascensions to allow all the stars to be observed on the same night. By such symmetrical arrangement all the instrumental errors will be eliminated from the final result for the value of α . I have partially discussed these observa-

tions. The general tendency is to confirm my previous conclusions, although, in some of the series, I note that the personality depending upon magnitude seems to have diminished. I do not doubt that this has arisen, unconsciously on part of the observers, from the fact that the observer, being warned of his possible tendency to such an error, has endeavoured to correct it. Still the general result is to show that in chronographic observing there is a tendency to make the R.A. of faint stars greater than by the Eye and Ear method.

I have also received some series of observations made after the manner which I suggested to Prof. BAKHUYZEN, Prof. LEWIS BOSS, and others, viz., to place a wire gauze screen over the object glass during the transit of a star across the first half of the wires, and to remove it whilst the star transits the second half of the wires, reversing the order in observation of the next star. I had afterwards the pleasure of witnessing the experiment at Leiden, where the result was perfectly successful. A 5th to 6th-magnitude star so observed appeared exactly like an 8th-magnitude star, not only as to brilliancy, but also as to general appearance, disk, &c., the interference rays being lost by the illumination of the field; nor could the most experienced observer have decided whether an 8th-magnitude star or a 5th to 6th-magnitude star reduced in brilliancy was being observed. The general tendency of the results was to show that, with chronographic observation, at least, the observer made the R.A. of the fainter stars too great.

However, from the fact that when an observer is conscious of a particular systematic error of observation he is apt to have a personality different from that which he had before he was aware of this error, I have, after much consideration, thought it best to employ the magnitude corrections which I have deduced from the Heliometer observations made simultaneously with the meridian observations, and without suspicion on the part of the meridian observers or of myself that such errors existed. The observations of the test stars I propose to discuss in a future memoir, together with the results of a series of observations, connected with personal equation, which are now being carried out at the Royal Observatory, Cape of Good Hope.

The reader who may desire to examine the details of the investigations, the results of which are given in the present chapter, is referred to the O.M.

CHAPTER X.

DEDUCTION OF THE DEFINITIVE MEAN AND APPARENT PLACES OF THE
COMPARISON STARS.

In the last chapter the systematic corrections depending upon magnitude, which have to be applied to the Preliminary Catalogue, were determined. The next step was to apply to the Preliminary Catalogue the corrections so determined. This having been done, a series of star places was obtained forming an Approximate Standard Catalogue, free from systematic error depending upon magnitude.

The original observations at each observatory were then corrected for proper motion to 1877·0*, and compared individually with the places of the Approximate Standard Catalogue.

The result of this comparison is given in the following Table.

As a matter of convenience the results are given in arc, where C—O indicates the excess of the right ascension of the Approximate Standard Catalogue over the corresponding right ascension at each Observatory.

* The proper motions, as derived from a discussion of all the old observations, are as follow :—

	In R.A.	In Dec.		In R.A.	In Dec.		In R.A.	In Dec.
<i>b</i>	^s ·0000	—·050	<i>e</i>	^s —·0043	·000	<i>l</i>	^s ·0000	—·016
<i>a</i>	—·0020	—·031	<i>δ</i>	—·0050	—·052	<i>λ</i>	+·0232	+·256
<i>c</i>	—·0013	·000	<i>ε</i> _o	—·0045	·000	<i>η</i>	—·0043	·000
<i>β</i>	—·0086	—·006	<i>f</i>	—·0080	·000	<i>σ</i>	—·0062	·000
<i>μ</i>	·0000	—·043	<i>g</i>	—·0015	—·030	<i>τ</i>	·0000	—·039
<i>d</i>	·0000	—·032	<i>ψ</i> ₁	+·0245	—·020			
<i>γ</i>	·0000	—·050	<i>k</i>	—·0050	·000			

The proper motions of the stars *α*, *θ*, *h*, *i*, *ζ*, *m*, *q*, *r* appear to be insensible.

TABLE I.

Table of Difference (in arc) between the General Mean R.A. corrected for Magnitude, and the R.A. of each particular Observatory not so corrected.

Star.	m.	Königsberg.		Melbourne.		Pulkowa.		Leipzig.		Greenwich.		Berlin.		Leiden.	
		C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.
a	1.5	-0.08	5	-0.15	3	+0.56	3	-1.08	4	-0.05	10	+0.66	4	-1.10	12
b	2.2	+0.36	6	-0.39	3	-0.21	6	+0.44	3	-0.11	5	-0.05	4	-1.34	4
c	5.0	+0.65	7	-1.71	3	-0.02	5	+1.10	3	-1.31	5	-0.63	2	-2.78	4
d	1.5	-0.08	6	+0.08	3	-0.05	5	-0.33	2	-0.08	5	+0.41	4	-1.07	7
e	3.2	-0.50	6	+0.36	3	-0.17	6	-0.51	4	-0.23	5	-1.04	3	-1.95	2
f	3.5	-0.80	6			-1.28	5	-0.60	2	-0.63	5	-0.11	3	-1.77	2
g	3.0	-1.77	6	-0.84	3	-0.84	3	+0.66	1	+0.30	5	-0.36	2	-1.34	3
h	4.0	-1.40	5	-0.33	3	-0.21	2	-0.74	2	-0.96	6	-0.89	1		
i	3.0	-0.75	4	+0.05	4	-1.14	5	-0.93	2	-0.51	5	-0.63	3	-1.19	4
j	4.5	-0.27	5	+0.02	3	-0.12	5	+3.66	2	-2.34	4	-0.92	2	-2.51	3
k	2.5	-1.16	5	-0.05	4	-0.90	2	+1.88	1	+0.30	4	+0.53	2		
l	3.3	-1.59	4	-0.02	3	-1.10	6	+1.11	2	-1.25	4	-0.17	5	-1.80	4
m	0.0	-0.56	4	-0.18	4			+0.20	1	-0.68	5	+0.87	2	-0.32	2
n	1.7	-1.29	5	-0.06	3	-1.26	4	-0.33	2	+0.12	5	+0.17	4	-1.43	2
o	0.7	-0.44	3	-0.27	4	-0.84	3	+0.09	2	+0.32	9	+0.42	3	-0.72	1
p	3.0	-0.59	4	-0.63	3	-1.08	3	-0.51	2	-0.89	4	+0.02	3	-1.47	4
q	3.0	-1.10	7	-0.30	3	-0.57	4	+0.36	3	-0.38	5	-0.50	5	-1.97	2
r	3.5	-1.76	5	-0.65	3	-0.60	3	+1.16	2	-1.07	5	-0.60	3	-2.37	2
s	3.5	-0.72	4	-0.99	3	-0.50	4	+0.48	2	-1.30	5	-0.15	3	-1.46	3
t	4.2	-0.84	8	-0.57	3	-0.77	3	+0.44	1	-1.43	5	-0.32	3	-2.25	3
u	2.3	-0.30	7	-0.51	3	-1.08	5	+1.59	3	-0.81	5	+0.14	3	-1.47	4
v	2.0	-0.39	8	-0.38	4	-1.02	3	+1.58	1	-0.21	7	+0.38	4	-0.81	2
w	2.5	-0.18	7	-0.59	3	-0.48	5	+0.75	2	-0.83	4	+0.20	4	-1.40	7
x	2.5	-0.84	7	-0.09	3	-0.03	4	-1.04	2	-0.38	5	-0.03	3	-1.23	4
y	3.0	-0.56	7	-0.53	3	-1.13	3	-1.13	1	-0.47	5	-0.15	4	-1.04	2
z	2.0	-0.78	7	-0.57	3	-1.32	1	-1.32	3	-0.30	6	+0.26	4	-1.28	7

Table of Difference, etc.—continued.

Star.	m.	Paris.		Washington.		Cambridge, U.S.		Cordoba.		Oxford.		Albany, U.S.	
		C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.	C—O.	No. of Obs.
<i>a</i>	1.5	+0.05	5	"		—0.59	6	—2.18	5	—0.30	7	—0.74	9
<i>b</i>	2.2	—1.97	5	—2.10	3	—1.73	6	—1.16	5	+0.24	7	—1.55	8
<i>a</i>	5.0	+0.11	5	—2.96	3	—4.56	3	—2.57	5	+0.83	5	—1.80	9
<i>c</i>	1.5	—1.01	6			—0.98	6	—0.54	7	—0.59	8	—1.38	9
<i>β</i>	3.2	—0.44	5	—2.64	3	—1.76	6	—0.92	5	+0.72	6	—1.70	9
<i>μ</i>	3.5			—2.60	3			—1.05	4	—0.36	5	—1.53	8
<i>d</i>	3.0	—0.81	6	—2.24	3	—2.16	6	—0.96	5	+0.11	4	—1.49	8
<i>γ</i>	4.0	—0.69	3	—2.84	3	—2.19	6	—1.34	5	+0.02	4	—2.60	8
<i>e</i>	3.0	+2.22	1	—1.64	3	—2.28	6	—2.01	5	+1.02	3	—1.89	8
<i>δ</i>	4.5			—3.24	3	—2.09	6	—1.80	5	—0.54	3	—1.25	10
<i>f</i>	2.5	+1.29	2	—3.38	3	—1.38	6	—1.25	5	—0.83	2	—0.95	8
<i>g</i>	3.3			—2.61	3	—1.56	6	—0.92	5	+1.34	1	—1.61	8
<i>ψ</i>	0.0	+2.30	1	—1.76	3	—0.86	6	—1.46	5	—0.53	4	—0.60	9
<i>h</i>	1.7	—0.36	3	—1.67	3	—1.44	5	—0.24	5	+1.08	5	—1.20	8
<i>i</i>	0.7	+0.31	1	—1.89	3	—0.87	6	—0.70	5	+1.49	5	—1.16	8
<i>k</i>	3.0			—2.24	3	—2.69	6	—0.80	5	+1.62	3	—1.47	8
<i>l</i>	3.0	—0.15	3	—2.75	3	—1.73	6	—0.38	5	+0.89	4	—1.50	8
<i>λ</i>	3.5	+0.06	3	—3.84	3	—2.07	6	—1.10	5	+0.78	4	—1.80	8
<i>ζ</i>	3.5			—2.10	3	—2.79	6	—1.70	5	+0.51	3	—2.12	10
<i>η</i>	4.2	+0.59	4	—3.32	2	—2.52	6	—1.49	5	+0.93	7	—1.68	7
<i>m</i>	2.3	—0.06	4	—2.31	3	—0.51	6	—1.29	5	+0.90	7	—1.98	8
<i>n</i>	2.0	+0.98	1	—2.07	4	—1.65	6	—1.04	5	+0.66	8	—1.64	9
<i>q</i>	2.5	—0.27	5	—2.54	3	—1.50	6	—1.01	4	—0.08	9	—1.10	9
<i>r</i>	2.5	+0.42	3	—2.69	3	—1.97	6	—0.89	5	—0.35	7	—1.58	9
<i>s</i>	3.0	+0.23	2	—3.03	3	—1.25	6	—1.58	6	+1.44	7	—2.27	8
<i>t</i>	2.0	+0.44	6	—2.63	3	—0.80	6	—0.63	5	+0.18	7	—1.10	9

It will be observed that the stars ϵ and θ are omitted. The former is a close double-star, and probably the personal error of its observation would be different from that of a single star of the same magnitude as that of either of its components. The star θ was not observed at many of the Observatories. The places of the stars ϵ and θ will be afterwards determined from the Heliometric triangulation.

If we assume that the differences of Table I. are due to

- (1) A systematic correction due chiefly to the system of clock stars employed, or to instrumental errors which have the effect of producing an error common to all the stars of the zone, but different for each Observatory.
- (2) A personal error depending on the magnitude and the square of the magnitude, also different for each Observatory.

Then we may represent the true R.A. of any star in any series of observations by

$$O + a + mb + m^2c.$$

where O is the observed R.A., m the difference of the star's magnitude from the standard magnitude ($4\frac{1}{2}$); and a , b , and c are constants to be determined.

If we put C = the Right Ascension of our Approximate Standard Catalogue, then each star observed at any Observatory furnishes an equation of the form

$$a + mb + m^2c = C - O \quad (1).$$

where the value of $C - O$ may be taken from the above table.

The observations of all the stars at the same Observatory may then be combined (having regard to the number of observations of each) to find the most probable value of a , b , and c for that Observatory.

I have carried out this investigation.

When the number of observations at a particular Observatory is nearly the same for all the stars, as is the case at Königsberg, Melbourne, Washington, Cambridge, U.S., Cordoba and Albany, I have given to the equations from each star an equal weight (1). But at Pulkowa, Leipzig, Greenwich, Berlin, Leiden, Paris, and Oxford, where some of the stars were observed much more frequently than others, a weight was given to the equations for each star equal to the number of observations of that star.

Forming equations in this way on the type of Equation (1) and treating them by the method of least squares, having regard to the weights above-mentioned, the following normal equations and values of a , b , and c were obtained:—

Königsberg.

$$\begin{array}{rcl}
 26^{\circ}0a + 71^{\circ}1b + 226^{\circ}9c & = & -17^{\circ}74 \\
 71^{\circ}1 + 226^{\circ}9 + 784^{\circ}3 & = & -50^{\circ}16 \\
 226^{\circ}9 + 784^{\circ}3 + 3006^{\circ}1 & = & -155^{\circ}71
 \end{array}
 \qquad
 \begin{array}{l}
 a = -''343 \\
 b = -''246 \\
 c = ''0382.
 \end{array}$$

Melbourne.

$$\begin{array}{rcl}
 25^{\circ}0a + 67^{\circ}6b + 214^{\circ}6c & = & -9^{\circ}30 \\
 67^{\circ}6 + 214^{\circ}6 + 741^{\circ}4 & = & -29^{\circ}76 \\
 214^{\circ}6 + 741^{\circ}4 + 2854^{\circ}8 & = & -108^{\circ}09
 \end{array}
 \qquad
 \begin{array}{l}
 a = -''100 \\
 b = -''0233 \\
 c = -''0242.
 \end{array}$$

Pulkowa.

$$\begin{array}{rcl}
 96^{\circ}7a + 282^{\circ}3b + 913^{\circ}3c & = & -59^{\circ}39 \\
 282^{\circ}3 + 920^{\circ}0 + 3230^{\circ}5 & = & -169^{\circ}87 \\
 913^{\circ}3 + 3230^{\circ}5 + 12120^{\circ}7 & = & -516^{\circ}67
 \end{array}
 \qquad
 \begin{array}{l}
 a = +''323 \\
 b = -''0758 \\
 c = +''01351.
 \end{array}$$

Leipzig.

$$\begin{array}{rcl}
 54^{\circ}5a + 149^{\circ}7b + 475^{\circ}1c & = & 13^{\circ}76 \\
 149^{\circ}7 + 477^{\circ}5 + 1667^{\circ}8 & = & 60^{\circ}07 \\
 475^{\circ}1 + 1667^{\circ}8 + 6259^{\circ}1 & = & 258^{\circ}62
 \end{array}
 \qquad
 \begin{array}{l}
 a = +''0274 \\
 b = -''0276 \\
 c = +''0113.
 \end{array}$$

Greenwich.

$$\begin{array}{rcl}
 135^{\circ}9a + 357^{\circ}5b + 1116^{\circ}5c & = & -71^{\circ}09 \\
 357^{\circ}5 + 1121^{\circ}2 + 3853^{\circ}5 & = & -252^{\circ}95 \\
 1116^{\circ}5 + 3853^{\circ}5 + 14212^{\circ}2 & = & -965^{\circ}76
 \end{array}
 \qquad
 \begin{array}{l}
 a = -''0169 \\
 b = +''0235 \\
 c = -''01185.
 \end{array}$$

Berlin.

$$\begin{array}{rcl}
 81^{\circ}6a + 217^{\circ}0b + 654^{\circ}8c & = & -4^{\circ}45 \\
 217^{\circ}0 + 664^{\circ}1 + 2177^{\circ}5 & = & -42^{\circ}46 \\
 654^{\circ}8 + 2177^{\circ}5 + 7627^{\circ}3 & = & -188^{\circ}24
 \end{array}
 \qquad
 \begin{array}{l}
 a = +''0928 \\
 b = -''0396 \\
 c = +''00087.
 \end{array}$$

Leiden.

$$\begin{array}{rcl}
 90^{\circ}8a + 231^{\circ}1b + 691^{\circ}7c & = & -131^{\circ}29 \\
 231^{\circ}1 + 691^{\circ}8 + 2319^{\circ}9 & = & -379^{\circ}65 \\
 691^{\circ}7 + 2319^{\circ}9 + 8509^{\circ}5 & = & -1256^{\circ}12
 \end{array}
 \qquad
 \begin{array}{l}
 a = -''0602 \\
 b = -''0196 \\
 c = -''00452.
 \end{array}$$

Paris.

$$\begin{array}{ll} 73.4a + 196.4b + 605.6c = -12.01 & a = +0.625 \\ 196.4 + 609.9 + 2100.6 = -28.88 & b = -0.692 \\ 605.6 + 2100.6 + 7911.4 = -56.54 & c = +0.1288. \end{array}$$

Washington.

$$\begin{array}{ll} 24.0a + 68.1b + 222.3c = -61.09 & a = -1.677 \\ 68.1 + 222.3 + 777.5 = -182.35 & b = -0.292 \\ 222.3 + 777.5 + 2995.5 = -612.31 & c = -0.0042. \end{array}$$

Cambridge, U.S.

$$\begin{array}{ll} 25.0a + 67.6b + 214.6c = -43.93 & a = -0.516 \\ 67.6 + 214.6 + 741.4 = -137.11 & b = -0.260 \\ 214.6 + 741.4 + 2854.8 = -482.12 & c = -0.0626. \end{array}$$

Cordoba.

$$\begin{array}{ll} 26.0a + 71.1b + 226.9c = -31.01 & a = -0.964 \\ 71.1 + 226.9 + 784.3 = -90.95 & b = +0.0978 \\ 226.9 + 784.3 + 3006.1 = -313.13 & c = -0.0569. \end{array}$$

Oxford.

$$\begin{array}{ll} 135.8a + 353.5b + 1079.7c = +53.41 & a = -0.103 \\ 353.5 + 1076.2 + 3633.2 = +158.99 & b = +0.277 \\ 1079.7 + 3633.2 + 13262.9 = +520.24 & c = -0.0283. \end{array}$$

Albany, U.S.

$$\begin{array}{ll} 26.0a + 71.1b + 226.9c = -39.69 & a = -0.762 \\ 71.1 + 226.9 + 784.3 = -116.33 & b = -0.345 \\ 226.9 + 784.3 + 3006.1 = -381.75 & c = +0.0205. \end{array}$$

Computing the value of $mb + m^2c$ for each value of m with the respective values of b and c as determined by the preceding discussion, we find the following systematic corrections for magnitude which should be applied to the Right Ascensions of each Observatory.

TABLE II.

Corrections to the Right Ascensions depending upon Magnitude.

<i>Observations by Eye and Ear.</i>						
Value of <i>m</i> .	1	2	3	4	5	Average Weight of R.A.
Corresponding Star Magnitude.	5.5	6.5	7.5	8.5	9.5	
Konigsberg ...	-0".21	-0".34	-0".39	-0".37	-0".28	1.70
Pulkowa ...	-0.62	-0.98	-1.06	-0.87	-0.41	0.71
Leipzig ...	-0.16	-0.10	+0.19	+0.70	+1.45	0.17
Paris ...	-0.56	-0.87	-0.92	-0.71	-0.24	0.39
Oxford ...	+0.25	+0.44	+0.58	+0.66	+0.68	0.64
<i>Observations by Chronograph.</i>						
Melbourne ...	-0".05	-0".14	-0".29	-0".48	-0".72	1.60
Greenwich ...	+0.12	0.00	-0.36	-0.96	-1.79	0.80
Berlin ...	-0.39	-0.76	-1.11	-1.45	-1.76	2.04
Leiden ...	-0.24	-0.57	-1.00	-1.51	-2.11	2.36
Washington ...	-0.30	-0.60	-0.91	-1.24	-1.57	0.93
Cambridge, U.S.	-0.33	-0.77	-1.35	-2.05	-2.88	1.32
Cordoba ...	+0.04	-0.03	-0.22	-0.52	-0.93	3.59
Albany ...	-0.32	-0.61	-0.85	-1.05	-1.21	2.69

In the original paper which I published, "On the Results of Meridian Observations of the Mars Comparison Stars" (*Monthly Notices*, vol. xxxix. p. 122), I wrote as follows:—

"It is remarkable, if confirmed by more extended experiment, that, as a rule, when chronographic registration is employed, the required correction to the observed Right Ascensions of faint stars is —, and where Eye and Ear observations are employed, it is +. It is most satisfactory to find that this

law coincides with the explanation I attempted to give as to the physical cause of the personality in question, viz. that in chronographic registration the tendency will be to press the key too late for faint stars, because their small neat disks make them appear farther from the web than a bright star with its larger disk and rings at the same angular distance from the web.

"I think it, therefore, not improbable that there may still remain systematic errors, depending on magnitude, in the Right Ascensions which have been deduced in the present discussion. The comparison with the Heliometer measures will, however, probably settle the question."

It was thus with no small degree of interest that I looked forward to the results of this discussion, and the remarks just quoted explain why, in the preceding Table, I have shown the results for the Eye and Ear observations separately from those made by Chronograph. In the first place, however, taking the mean of all these results, having regard to the weights, we obtain the following mean corrections:—

For Magnitude	5.5	6.5	7.5	8.5	9.5
Correction	—0".20	—0".41	—0".64	—0".92	—1".21

results which agree almost precisely with those determined by the Heliometer, whence, of course, they were originally derived.

The very small difference is explained as follows:—

- (1) Because of the introduction of the Albany observations which were not included in the preliminary catalogue.
- (2) Because no rigid computation of the weights of the magnitude corrections has been made, it having been assumed that the magnitude corrections will have the same proportional weight as the mean of the weights of an R.A. observation in each series.
- (3) Because I have somewhat arbitrarily assumed that the form of a curve of magnitude correction may, in each case, be represented with sufficient accuracy by a parabolic curve, i.e. may be expressed in terms of m and m^2 . I was anxious to avoid the employment of the graphic method, because the drawing of the curves

then becomes, to a certain extent, a matter of judgment or opinion. From the form of Prof. BAKHUYZEN's curves (*Monthly Notices*, vol. xxxix. p. 122) it is sufficiently evident that from magnitude 6 to magnitude 8·5 a parabolic curve will represent the true curve in every case without sensible error. Now all the stars which were actually employed for comparison with *Mars* lie within the range of magnitude in question, with the single exception of the star *i* which was compared with *Mars* on August 29 and September 2. On both these occasions, however, the factor in R.A. is so very small, the star being nearly due North of *Mars*, that any questions as to minute errors of its R.A. need not be considered. I believe, therefore, that upon the whole I have arrived at an unprejudiced determination of the absolute corrections of R.A. from magnitude 6 to magnitude 8·5.

If now we take the mean of the Eye and Ear observations, and the mean of the Chronograph observations, having regard to the weight in both cases, we find

Corrections due to Magnitude	5·5	6·5	7·5	8·5	9·5
Eye and Ear	−0"24	−0"37	−0"38	−0"27	−0"05
Chronograph	−0'19	−0'42	−0'72	−1'08	−1'49

Here we have two rather remarkable results:

- (1) For observations by Eye and Ear the correction for magnitude is the same for all magnitudes within the probable limits of error of observation.
- (2) For *all* the Chronograph observations the Right Ascensions of the faint stars are too great.

But these two results do not rest simply on a mean, but they apply to the results of each separate Observatory as shown in the following Table:—

Eye and Ear Observations.

	Deduced Correction for 6th Magnitude.	Deduced Correction for 8·5 Magnitude.	Difference 6th Mag.-8·5 Mag.
Königsberg	-0"28	-0"37	+0"09
Pulkowa	-0"80	-0"87	+0"07
Leipzig	-0"14	+0"70	-0"84
Paris	-0"72	-0"71	-0"01
Oxford	+0"35	+0"66	-0"31

Now each of those differences is equal to 0 nearly within the probable error of its determination. The observations at Leipzig were very limited in number, eight of the stars were only observed once, and the mean error of one observation is $\pm 1''\cdot76$, so that even so large a difference as $0''\cdot\pm 84 (=0\cdot056^{\text{sec}})$ is quite within the limit of error.

The second result is corroborated without a single exception as follows:—

Chronographic Observations.

	Deduced Correction for 6th Magnitude.	Deduced Correction for 8·5 Magnitude.	Difference 6th Mag.-8·5 Mag.
Melbourne	-0"08	-0"48	+0"40
Greenwich	+0"09	-0"96	+1"05
Berlin	-0"57	-1"45	+0"88
Leiden	-0"39	-1"51	+1"12
Washington	-0"45	-1"24	+0"79
Cambridge, U.S. ...	-0"54	-2"05	+1"51
Cordoba	+0"02	-0"52	+0"54
Albany	-0"48	-1"05	+0"57

These facts bring us face to face with a very unpleasant conclusion, viz. that in adopting the chronographic method of observation, astronomers, whilst gaining a greater *apparent* exactitude, have really introduced a serious new systematic error, whose effects, if not studied and eliminated, may be very

disastrous to the progress of science. For example, BESSEL's zones, which were observed about 1825, will very properly be used, in due course of time, as a foundation for discussions on precession, proper motion, and the motion of the Solar System through space.* These zones were observed by Eye and Ear, and if we may assume the observers at Königsberg, Pulkowa, Paris, and Oxford to be typical of all Eye and Ear observers, then BESSEL's zones would be free from errors depending on magnitude.† But if all observers by the chronographic method have errors depending on magnitude like the observers at Melbourne, Greenwich, Berlin, Leiden, Washington, Cambridge, U.S., Cordoba, and Albany,‡ we should from modern star catalogues deduce quite a different value of precession, according as we employ faint or bright stars. We should also obtain erroneous results for proper motion which might lead to wrong conclusions as to the motion of the Solar System through space, as to the distribution of the stars of different magnitude, and as to the dimensions of the Sidereal System.

I propose to deal with this subject more fully in a future memoir, and shall meanwhile proceed to apply the results to form our Standard Catalogue. The following Table gives the corrections for each Observatory derived from the equations pp. 76, 77.

* They have already been used by NYRÉN in his determination of the constant of Precession.

† ARGELANDER, who observed by Eye and Ear, showed, by observing variable stars, that his personal error depending on magnitude was insensible.

‡ According to a few observations shown to me by Dr. WAGNER at Pulkowa in the beginning of 1879, when a bright star and a neighbouring faint star had been frequently observed both by Eye and Ear and Chronograph about the same epoch, the personal error of that distinguished observer indicated a small error in the reverse direction, viz. the R. A. of the faint star was made greater, relative to the brighter star, by the Eye and Ear method than by the Chronographic. The observations were not very numerous, and besides Dr. WAGNER had undertaken a long course of training with artificial stars with this possible error specially in view. Doubtless in this way he conquered the natural tendency which appears to exist in most observers, if we may judge from the very strong evidence from above.

TABLE III.

Observatory.	Systematic Correction.	Corrections depending upon Magnitude.										
		4.5	5.0	5.5	6.0	6.5	7.0	7.5	8.0	8.5	9.0	9.5
Königsberg ...	^s - '023	^s ± '000	^s - '008	^s - '014	^s - '019	^s - '023	^s - '025	^s - '026	^s - '026	^s - '025	^s - '022	^s - '018
Pulkowna ...	+ '022	'000	- '023	- '042	- '056	- '065	- '070	- '071	- '067	- '058	- '045	- '027
Leipzig ...	+ '002	'000	- '007	- '011	- '011	- '007	+ '001	+ '013	+ '028	+ '047	+ '070	+ '096
Paris ...	+ '042	'000	- '021	- '038	- '050	- '058	- '062	- '061	- '056	- '047	- '033	- '016
Oxford...	- '007	'000	+ '009	+ '017	+ '024	+ '029	+ '034	+ '038	+ '041	+ '043	+ '045	+ '045
Melbourne ...	- '007	'000	- '001	- '003	- '005	- '009	- '014	- '019	- '025	- '032	- '040	- '048
Greenwich ...	- '011	'000	+ '006	+ '008	+ '006	'000	- '010	- '024	- '042	- '064	- '090	- '119
Berlin ...	+ '062	'000	- '013	- '026	- '038	- '050	- '062	- '074	- '086	- '097	- '107	- '117
Leiden...	- '040	'000	- '007	- '016	- '026	- '038	- '051	- '066	- '082	- '100	- '120	- '141
Washington ...	- '112	'000	- '010	- '020	- '030	- '040	- '050	- '061	- '071	- '082	- '093	- '104
Cambridge ...	- '034	'000	- '010	- '022	- '036	- '052	- '070	- '090	- '112	- '136	- '163	- '192
Cordoba ...	- '064	'000	+ '003	+ '003	+ '001	- '002	- '008	- '015	- '024	- '035	- '048	- '062
Albany...	- '050	'000	- '011	- '022	- '032	- '041	- '049	- '057	- '064	- '070	- '076	- '081

Applying these corrections to separate results of each Observatory (corrected for proper motion to 1877.0) we obtain the following :—

TABLE IV.
Right Ascensions at each Observatory, Corrected for Systematic Error and Magnitude.

	a		b		α		c		β		μ		d		γ		e	
	^s	^p	^s	^p	^s	^p	^s	^p	^s	^p	^s	^p	^s	^p	^s	^p	^s	^p
Königsberg ...	.980	1.53	.706	1.79	.381	2.04	.879	1.79	.501	1.79	.894	1.79	.893	1.79	.576	1.53	.882	1.26
Melbourne ...	.015	1.59	.785	1.59	.524	1.59	.899	1.59	.465	1.59			.854	1.59	.514	1.59	.852	2.03
Pulkowa ...	.946	0.55	.746	1.05	.461	0.89	.885	0.89	.481	1.05	.930	0.89	.831	0.55	.509	0.37	.908	0.89
Leipzig ...	.080	0.33	.746	0.25	.490	0.25	.929	0.17	.572	0.33	.960	0.17	.795	0.09	.629	0.17	.958	0.17
Greenwich ...	.015	1.44	.769	0.77	.422	0.77	.916	0.77	.490	0.77	.879	0.77	.809	0.77	.520	0.92	.880	0.77
Berlin ...	.997	2.48	.787	2.48	.452	1.38	.913	2.48	.569	1.97	.873	1.97	.788	1.38	.555	0.74	.911	1.97
Leiden ...	.024	5.35	.783	2.62	.469	2.62	.921	3.91	.535	1.49	.886	1.49	.807	2.09			.854	2.62
Paris ...	.006	0.55	.890	0.55	.484	0.55	.975	0.65	.529	0.55			.859	0.65	.572	0.33	.714	0.11
Washington .			.761	0.93	.446	0.93			.516	0.93	.881	0.93	.800	0.93	.536	0.93	.817	0.93
Cambridge U.S.	.986	1.32	.799	1.32	.543	0.70	.911	1.32	.501	1.32			.844	1.32	.507	1.32	.909	1.32
Cordoba ...	.099	3.59	.786	3.59	.510	3.59	.889	4.46	.495	3.59	.872	3.07	.809	3.59	.521	3.59	.936	3.59
Oxford ...	.054	0.85	.785	0.85	.448	0.62	.972	0.97	.501	0.73	.948	0.62	.848	0.50	.566	0.50	.844	0.38
Albany ...	.984	2.94	.786	2.69	.452	2.94	.925	2.94	.514	2.94	.878	2.69	.816	2.69	.584	2.69	.897	2.69

Right Ascensions at each Observatory, etc.—continued.

	δ		f		g		ψ		h		i		k		l		λ	
	s	p	s	p	s	p	s	p	s	p	s	p	s	p	s	p	s	p
Königsberg ...	.831	1.53	.444	1.53	.887	1.26	.768	1.25	.778	1.53	.707	0.97	.985	1.26	.551	2.04	.467	1.53
Melbourne ...	.850	1.59	.397	2.03	.801	1.59	.759	2.03	.726	1.59	.720	2.03	.011	1.59	.521	1.59	.410	1.59
Pulkowa ...	.843	0.89	.427	0.37	.855	1.05			.782	0.73	.758	0.55	.018	0.55	.516	0.73	.394	0.55
Leipzig ...	.686	0.17	.293	0.09	.780	0.17	.743	0.09	.751	0.17	.698	0.17	.044	0.17	.518	0.25	.352	0.17
Greenwich ...	.913	0.63	.374	0.63	.868	0.63	.788	0.77	.721	0.77	.686	1.32	.019	0.63	.517	0.77	.417	0.77
Berlin ...	.874	1.38	.380	1.38	.822	2.94	.758	1.38	.744	2.48	.727	1.97	.982	1.97	.548	2.94	.415	1.97
Leiden ...	.865	2.09			.834	2.62	.735	1.49	.760	1.49	.708	0.80	.987	2.62	.552	1.49	.435	1.49
Paris ...			.309	0.22			.643	0.11	.749	0.33	.704	0.11			.518	0.33	.381	0.33
Washington .	.869	0.93	.478	0.93	.825	0.93	.759	0.93	.701	0.93	.711	0.93	.971	0.93	.537	0.93	.472	0.93
Cambridge ...	.800	1.32	.403	1.32	.797	1.32	.777	1.32	.756	1.12	.720	1.32	.050	1.32	.518	1.32	.391	1.32
Cordoba ...	.866	3.59	.426	3.89	.807	3.59	.787	3.59	.688	3.59	.697	3.59	.969	3.59	.473	3.59	.384	3.59
Oxford ...	.932	0.38	.497	0.26	.774	0.13	.782	0.50	.683	0.62	.617	0.62	.918	0.38	.499	0.50	.381	0.50
Albany ...	.811	3.17	.379	2.69	.825	2.69	.763	2.94	.730	2.69	.723	2.69	.982	2.69	.520	2.69	.423	2.69

Right Ascensions at each Observatory, etc.—continued.

	ζ		η		m		n		q		r		s		t	
	s	p	s	p	s	p	s	p	s	p	s	p	s	p	s	p
Königsberg ...	.316	1.26	.281	2.27	.019	2.04	.801	2.27	.322	2.04	.659	2.04	.237	2.04	.268	2.04
Melbourne ...	.351	1.59	.268	1.59	.061	1.59	.830	2.03	.376	1.59	.636	1.59	.259	1.59	.284	1.59
Pulkowa ...	.305	0.73	.292	0.55	.072	0.89	.846	0.55	.342	0.89	.605	0.73	.276	0.55	.307	0.19
Leipzig ...	.315	0.17	.301	0.09	.940	0.25	.921	0.09	.311	0.17	.723	0.17	.340	0.09	.169	0.25
Greenwich ...	.351	0.77	.282	0.77	.083	0.77	.824	1.06	.392	0.63	.655	0.77	.246	0.77	.271	0.92
Berlin ...	.303	1.97	.254	1.97	.042	1.97	.808	2.48	.345	2.48	.653	1.97	.248	2.48	.257	2.48
Leiden ...	.292	2.09	.274	2.09	.058	2.62	.797	1.49	.360	3.91	.642	2.62	.213	1.49	.269	3.91
Paris ...			.234	0.44	.032	0.44	.740	0.11	.356	0.55	.603	0.33	.216	0.22	.217	0.65
Washington .	.274	0.64	.295	0.93	.042	1.21	.807	0.93	.365	0.93	.668	0.93	.279	0.93	.285	0.93
Cambridge ..	.357	1.32	.259	1.32	.983	1.32	.845	1.32	.354	1.32	.678	1.32	.209	1.32	.229	1.32
Coedoba ...	.342	3.59	.267	3.59	.062	3.59	.824	3.07	.353	3.07	.638	3.59	.276	4.05	.238	3.59
Oxford ...	.317	0.38	.247	0.85	.011	0.85	.799	0.97	.390	1.08	.701	0.85	.185	0.85	.272	0.85
Albany ...	.344	3.17	.259	2.43	.082	2.69	.833	2.94	.332	2.94	.657	2.94	.289	2.69	.244	2.94

The following Table gives the mean of these Right Ascensions, and also the final Right Ascensions, having regard to weight:—

Right Ascension of the Mars Comparison Stars.

	Arithmetical Mean.	Mean having regard to Weight.	Difference.
	h m s	s	s
<i>a</i>	22 47 0'016	0'021	-0'005
<i>b</i>	47 37'779	37'779	± 0
<i>u</i>	51 39'468	39'467	+ 1
<i>c</i>	53 6'918	6'912	+ 6
β	56 5'513	5'511	+ 2
μ	58 2'900	2'885	+ 5
<i>d</i>	59 27'827	27'826	+ 1
γ	23 0 31'549	31'542	+ 7
<i>e</i>	0 57'874	57'889	- 15
δ	5 3'842	3'843	- 1
<i>f</i>	8 15'401	15'408	- 7
<i>g</i>	8 55'823	55'825	- 2
ψ , ...	9 26'755	26'766	- 11
<i>h</i>	11 14'736	14'726	+ 10
<i>i</i>	12 33'706	33'709	- 3
<i>k</i>	12 36'995	36'988	+ 7
<i>l</i>	14 28'522	28'523	- 1
λ	16 37'409	37'414	- 5
ζ	16 54'322	54'327	- 5
η	20 13'270	13'268	+ 2
<i>m</i>	21 41'037	41'048	- 11
<i>n</i>	22 38'821	38'819	+ 2
<i>q</i>	26 3'354	3'353	+ 1
<i>r</i>	29 5'655	5'652	+ 3
<i>s</i>	29 39'252	39'254	- 2
<i>t</i>	31 51'255	51'257	- 2

Strictly speaking, a new determination of weights should now be made, because the systematic error s , introduced into the discussion of the weights (*Monthly Notices*, vol. xxxix. p. 115) will, in some cases at least, be nearly eliminated by the corrections for magnitude which have been applied. But the results having regard to weight agree so closely with the simple arith-

metical mean that I do not think the small alterations in the weights, which might result from their rediscussion, would in any case sensibly affect the final Right Ascensions.

Now as to the Absolute System of Right Ascension. A considerable proportion of the Observatories have employed for clock error the places of the catalogue of 539 stars of the *Astron. Gesellschaft*, but the actual systematic corrections which we have obtained show that, besides the corrections for magnitude, and besides the difference in the systems of clock stars employed, there are large differences in the absolute Right Ascensions, common to all the stars under discussion, which probably are chiefly due to instrumental error. The *Mars* stars have from 9° to 14° South Declination, whilst most of the clock stars have North Declination: hence errors of the pivots, and lateral flexure (produced by buckling of the cube or yielding of one side of the horizontal axis more than another), may produce the errors in question. Besides these possible instrumental errors, the different position of the observer's head may produce an undetermined change in his personal equation.

It is for these reasons that I think it the safest plan to adopt the mean system of all the Observatories, there being no data for determining the relative probability of freedom of each Observatory from these errors.

In the preceding discussion the final places do not truly represent a mean system. The Albany observations were reduced to the system of the mean of twelve Observatories, as they were not included in the preliminary catalogue, so that they are not represented as regards absolute R.A. in the final mean; and besides this, in the preceding discussion, the systematic corrections are applied with their weights.

I therefore reduced the whole series again, applying the corrections for magnitude only, taking the mean, and thence deducing the systematic correction. The result showed that to reduce to a true mean system of the thirteen Observatories the preceding Right Ascensions require in the mean a correction of

$$+0^{\circ}014.$$

I do not propose to discuss further at present the question of the absolute Declinations of the stars. There will be soon so much more ample *matériel* for this discussion when Prof. Boss's new work on Standard Declinations

is published, and when all the series of meridian observations of *Mars* in 1877 have also been published, that I think the question can be with advantage deferred, particularly as the small correction, whatever it may be, does not affect our results for parallax, and it can be applied at the end of the discussion with as much convenience as now.

Adopting, then, a system of mean Declination, and the Right Ascensions found in the present chapter (increased by 0.014^{sec}) as definitive, we have the following places.

TABLE V.

Definitive Mean Places of the Mars Comparison Stars for 1877.0.

	Right Ascensions.		Declinations.	Corrections to Preliminary Catalogue.		
	Time.	Arc.		$\Delta\alpha$ (time).	$\Delta\alpha$ (arc).	$\Delta\delta$
	^h ^m ^s	[°] ['] ["]	[°] ['] ["]			
<i>a</i>	22 47 0.035	341 45 0.53	-12 16 12.10	-0.008	-0.12	+0.02
<i>b</i>	47 37.793	341 54 26.90	-12 50 34.41	- .022	-0.33	- .03
<i>a</i>	51 39.481	342 54 52.22	-11 47 20.86	- .070	-1.05	+ .06
<i>c</i>	53 6.926	343 16 43.89	-13 43 46.19	- .015	-0.23	- .05
<i>B</i>	56 5.525	344 1 22.88	-11 55 34.56	- .042	-0.63	.00
<i>μ</i>	58 2.899	344 30 43.49	-12 50 28.20	- .052	-0.78	+ .04
<i>d</i>	22 59 27.840	344 51 57.60	-11 6 3.46	- .036	-0.54	+ .05
<i>γ</i>	23 0 31.556	345 7 53.34	-13 23 27.35	- .045	-0.68	+ .04
<i>e</i>	0 57.903	345 14 28.55	-12 28 15.36	- .027	-0.41	+ .01
<i>δ</i>	5 3.857	346 15 57.86	-11 10 32.16	- .076	-1.14	- .02
<i>f</i>	8 15.422	347 3 51.33	-11 21 25.45	- .036	-0.54	.00
<i>g</i>	8 55.839	347 13 57.59	-12 14 4.98	- .048	-0.72	+ .03
<i>ψ</i>	9 26.780	347 21 41.70	- 9 45 27.18	+ .007	+0.11	+ .01
<i>h</i>	11 14.740	347 48 41.10	-12 23 4.21	- .026	-0.39	- .06
<i>i</i>	12 33.723	348 8 25.85	-10 16 58.08	± .000	±0.00	- .02
<i>k</i>	12 37.002	348 9 15.03	-12 50 33.67	- .041	-0.62	+ .03
<i>l</i>	14 28.537	348 37 8.06	-11 12 19.49	- .042	-0.63	+ .04
<i>λ</i>	16 37.428	349 9 21.42	-11 26 51.14	- .050	-0.75	+ .06
<i>ζ</i>	16 54.341	349 13 35.12	-10 3 33.22	- .037	-0.56	- .13
<i>η</i>	20 13.282	350 3 19.23	-10 42 37.03	- .061	-0.92	+ .03
<i>m</i>	21 41.062	350 25 15.93	-12 7 33.29	- .024	-0.36	- .04
<i>n</i>	22 38.833	350 39 42.50	- 9 56 33.38	- .017	-0.26	- .07
<i>q</i>	26 3.367	351 30 50.51	-11 40 39.35	- .035	-0.53	- .01
<i>r</i>	29 5.666	352 16 24.99	-11 14 5.02	- .029	-0.44	- .01
	29 39.268	352 24 49.02	- 9 26 42.12	- .030	-0.45	- .06
<i>t</i>	23 31 51.271	352 57 49.07	- 9 18 28.20	- .026	-0.39	+0.02

There still remain, however, the places of the stars ϵ and θ to be discussed. The distance of the star ϵ was measured from the stars

$g, e, f, h, \mu, \gamma, \delta$ and k .

These measures (O.M. Chap. IX. Tables III. and IV.) furnish the following equations for determining the place of ϵ_0 *:—

$$\begin{array}{rclcl}
 g-\epsilon_0 & -0.89\Delta\alpha -0.41\Delta\delta & = & -0.68+0.46\Delta S \\
 e-\epsilon_0 & +.97 & -.12 & = +.66+0.58 \\
 f-\epsilon_0 & -.46 & -.88 & = +.12+0.72 \\
 h-\epsilon_0 & -.96 & -.15 & = -.02+0.72 \\
 \mu-\epsilon_0 & +.98 & +.13 & = +.74+0.94 \\
 \gamma-\epsilon_0 & +.82 & +.54 & = -.24+0.75 \\
 \delta-\epsilon_0 & +.08 & -.99 & = +.53+0.73 & \frac{1}{2} \text{ weight} \\
 k-\epsilon_0 & -.97 & +.14 & = -.34+0.89 & \frac{1}{2} \text{ weight.}
 \end{array}$$

Correcting these equations for the errors of the places of the stars $g, e, f, h, \mu, \gamma, \delta$, and k as found in the preceding table, by adding to the right-hand side of the equation a number equal to the sum of the corresponding products of $\Delta\alpha$ and $\Delta\delta$ multiplied by the coefficients given on the left-hand side of the equations, and also substituting for ΔS its numerical value according to the 1st and 5th System, we obtain the following corrected equations.

	First System.	Fifth System.
$-0.89\Delta\alpha -0.41\Delta\delta$	$= -1.06$	$= -1.29$
$+ .97 \quad - .12$	$= +1.31$	$= +1.08$
$- .46 \quad - .88$	$= +0.08$	$= -0.10$
$- .96 \quad - .15$	$= -0.18$	$= -0.36$
$+ .97 \quad + .13$	$= +1.51$	$= +1.54$
$+ .82 \quad + .54$	$= +0.61$	$= +0.44$
$(\frac{1}{2} \text{ weight}) + .08 \quad - .99$	$= +0.82$	$= +0.64$
$(\frac{1}{2} \text{ weight}) - .97 \quad + .14$	$= -0.85$	$= -0.89$

By the method of least squares, and having regard to the weights, we obtain the normal equations

	First System.	Fifth System.
$4.960\Delta\alpha + 1.256\Delta\delta$	$= 4.763$	4.905
$1.256 \quad + 1.788$	$= 0.294$	0.601

* ϵ is a double star; ϵ_0 denotes the mean place of its two components.

The solution of which gives

First System.	Fifth System.	
"	"	"
$\Delta a_s = +1.117$	$+ 1.099$	Mean $+1.11 = 0.074$
$\Delta \delta_s = -0.619$	$- 0.435$	$- 0.53$

The substitution of these values in the original corrected equations gives the following residuals:—

	First System.	Fifth System.
$g-\epsilon_0$	$+0.31$	$+0.49$
$e-\epsilon_0$	$- .15$	$+ .04$
$f-\epsilon_0$	$- .06$	$- .03$
$h-\epsilon_0$	$- .81$	$- .63$
$\mu-\epsilon_0$	$- .50$	$- .53$
$\gamma-\epsilon_0$	$- .02$	$+ .23$
$\delta-\epsilon_0$	$- .12$	$- .12$
$k-\epsilon_0$	$- .33$	$- .24.$

To determine the place of θ we have the following equations from Tables III. and IV. of last Chapter:—

$$\begin{aligned}
 k-\theta & -0.71\Delta a -0.69\Delta \delta = 0.48+0.90\Delta S \\
 \gamma-\theta & + .90 - .37 = - .13+ .92 \\
 g-\theta & - .20 - .98 = - .54+ .95 \\
 h-\theta & - .48 - .87 = - .63+ .99 \\
 \epsilon_0-\theta & + .28 - .96 = - .14+ .78 \quad (\text{weight } 2).
 \end{aligned}$$

Correcting these equations as before for errors of the star places of k , γ , g , and h , and taking for errors of the place of ϵ_0 , $\Delta a_s = +1''.11$ and $\Delta \delta_s = -0''.62$ or $-0''.44$, as just determined, and substituting the values of ΔS in the two systems under discussion, we obtain the following corrected equations:—

	First System.	Fifth System.	
	"	"	
$k-\theta$	$-0.71\Delta a$	$-0.69\Delta \delta$	$= +0.14 + 0.12$
$\gamma-\theta$	$+ .90$	$- .37$	$= + .56 + .55$
$g-\theta$	$- .20$	$- .98$	$= - .65 - .61$
$h-\theta$	$- .48$	$- .87$	$= - .90 - .81$
$\epsilon_0-\theta$	$+ .28$	$- .96$	$= - .86 - .82 \text{ (weight } 2).$

By the method of least squares having regard to the weights, the resulting normal equations are—

$$\begin{array}{rcccl} & & \text{First} & \text{Fifth} & \\ & & \text{System.} & \text{System.} & \\ 1.736\Delta\alpha + 0.244\Delta\delta & = & +0.495 & = & +0.469 \\ 0.244 & + & 4.153 & = & +2.750 = +2.582, \end{array}$$

the solution of which gives—

$$\begin{array}{rcccl} & \text{First} & \text{Fifth} & & \\ & \text{System.} & \text{System.} & & \\ \Delta\alpha_\theta & = & +0.194 + 0.184 & \text{Mean} & +0.189 = 0.013 \\ \Delta\delta_\theta & = & +0.651 - 0.611 & & +0.63 \end{array}$$

The substitution of these values of $\Delta\alpha$ and δ in the original corrected equations gives the following residuals:—

	First System.	Fifth System.
$k-\theta$	-0.73	-0.67
$\gamma-\theta$	-0.63	-0.61
$g-\theta$	-0.03	-0.03
$h-\theta$	$+0.24$	$+0.19$
$\epsilon_0-\theta$	$+0.29$	$+0.28$

The final place of θ is therefore

		R.A.			Dec.		
		h	m	s	°	'	"
Place of θ in Preliminary Catalogue		23	7	21.580	14	3	46.44
$\Delta\alpha$ as above	...			+0.013	$\Delta\delta$		+0.63
Final Mean R.A. and Dec. ...	...	23	7	21.593	14	3	45.81
	R.A. in arc	346	50	23.90.			

It is also interesting to see how entirely the personal equation of the observers has been changed in observing the double-star ϵ .

Instead of a negative correction to the place of the preliminary catalogue amounting to -0.04^{sec} or -0.05^{sec} we have a correction of $+0.074^{\text{sec}}$, so that there is a discordance amounting to more than 0.1^{sec} of time. This fact

appears to show that double-stars should never be used for determining clock error.

The final place of the star ϵ_0 is therefore as follows:—

	R. A.			Dec.		
	h	m	s	°	'	"
Place of star ϵ_0 in Preliminary Catalogue	23	5	33.517	-12	36	1.66
$\Delta\alpha$			+0.074	$\Delta\delta$		- .53
	23	5	33.591		-12	36 2.19

The constants a , b , c , d , &c., for computing the corrections to apparent place for Greenwich-midnight, are given in the O.M., together with the actual corrections found by their means. These corrections were computed for each fifth day, and then rigidly interpolated for the epoch of each observation. The terms omitted in the *Nautical Almanac* Day Numbers (depending on 2 D, etc.) have not been included in the computation of the apparent places of the stars, because the same terms have been omitted in the computation of the apparent places of *Mars*.

CHAPTER XI.

FORMATION OF THE TABULAR PLACES OF *MARS*.

In the *Monthly Notices*, vol. xxxix. p. 54, I have described the method by which the tabular places of *Mars* were computed from LEVERRIER'S Tables, and by which the preliminary corrections of these Tables were ascertained.

In computing the tabular parallax, the compression of the Earth was assumed $\frac{1}{300}$.*

The tabular horizontal mean Solar parallax adopted was 8".80.

The O.M. contains the following Tables which may be useful for reference.

* Whence $\phi' = -7^{\circ} 52' 43.7''$ for Garrison.
and $= -7^{\circ} 56' 5.5''$ for Mars Bay.
and $\log. \rho = 9.99972$.

Table I. Apparent Geocentric R.A. and Dec. of *Mars*, to $\frac{1}{100}$ of a second of arc, for each alternate day from July 24 to October 12.

Table II. Apparent Geocentric R.A. and Dec. of *Mars*, for each $\frac{1}{100}$ of a day, during the actual periods of observation at Ascension, interpolated from the places of Table I.

Table III. Corrections to the R.A. and Dec. of *Mars* corresponding with an error of $+2''.45$ in Heliocentric Longitude.

Table IV. Values of $\log. \frac{1}{\Delta}^*$ for each $\frac{1}{10}$ of a day from July 30 to October 5.

CHAPTER XII.

THE HELIOMETER OBSERVATIONS OF *MARS* AND THEIR REDUCTION.

In the O.M. I have given complete details of all the observations of *Mars*, together with the results of the successive steps of their reduction. It will be sufficient here to give a single example of one of these groups by way of illustration.

* Where Δ is the distance of *Mars* from the Earth, the mean distance of the Earth from the Sun being reckoned 1.

1	2	3	4	5	6	7	8	9	10	11
Group. Date. Object.	Rotation Number.	Time by Chronometer Face.	Hour Angle of Middle Point.*	Heliometer Readings.				Resulting Distance in Terms of Micrometer Screw.	Refraction.	Instrumental Distance.
				Scale A.		Scale B.				
				Scale.	Screw.	Scale.	Screw.			
43	171	^h 19 ^m 15 ^s 37	^h -4 ^m 9 ^s 3	116	^r 3'920	31	^r 3'307	^r 169'485	^r .1978	^r 169'6828
July 31	172	19 39	-4 5'2	32	2'725	116	3'095	.512	.1858	.6978
Mars & n	173	22 12	-4 2'7	32	2'700	116	3'088	.530	.1791	.7091
	174	27 53	-3 57'0	116	3'964	31	3'245	.590	.1653	.7553

* Required for computing the refraction.

12	13	14	15	16	17	18	19	20	21
Rotation Number.	G. M. T. in Decimals of a Day.	Hour Angle of Planet.	Tabular Geocentric R.A. of Mars.	Tabular Geocentric Declination of Mars.	Tabular Par. in R. A.	Tabular Par. in Dec.	Star's App. R.A. Star's App. Dec. Correction of Tab. R.A. of Mars. Do. of Dec.	Computed Distance of Centre of Mars from Star.	Computed P. Angle of Centre of Mars from Star.
171	.48251	^h -4 ^m 11 ^s 42	351° 53' 30 ^{''} 87	-9° 46' 16 ^{''} 60	+17 ^{''} 69	+1 ^{''} 19	350° 40' 33 ^{''} 70 -9 56 12 ^{''} 22 Δα + 6 ^{''} 15 Δδ + 3 ^{''} 09	4364 ^{''} 53 65 ^{''} 16 65 ^{''} 55 66 ^{''} 41	82° 11' 11 11 11
172	.48331	7 40	31 ^{''} 70	16 ^{''} 76	+17 ^{''} 53	+1 ^{''} 14			
173	.48708	5 7	32 ^{''} 23	16 ^{''} 86	+17 ^{''} 42	+1 ^{''} 11			
174	.49100	-3 59 26	33 ^{''} 39	17 ^{''} 08	+17 ^{''} 18	+1 ^{''} 04			

Precisely similar details are given in the O.M. for each of the 343 groups of observations.

Column 13 gives the Greenwich Mean Time, computed in decimals of a day, for convenience in taking out the tabular place of the Planet, from Table II. of Chapter XI. O.M.

Column 14 gives the Hour Angle of the Planet, required for computing the parallax.

Column 15 gives the tabular Apparent Geocentric R.A. of the Planet, according to LEVERRIER'S Tables, interpolated for the instant of observation from Table II. of Chapter XI. O.M.

Column 16 gives similarly the tabular Apparent Geocentric Dec. of the planet.

Columns 17 and 18 give the tabular Parallax in R.A. and Dec. respectively, if Mean $\odot$ ' Parallax = $8''.80$.

Column 19 contains the Apparent R.A. and Dec. of the star of comparison, computed from the definitive data of Chapter X. In the same column will also be found the tabular corrections $\Delta\alpha$ and $\Delta\delta$, which are to be applied in R.A. and Dec. respectively, to LEVERRIER'S tabular places, to correct for an error of heliocentric longitude = $+2''.45$; the values of these corrections are interpolated from Table III. of Chapter XI. O.M.

Columns 20 and 21 give the tabular Apparent Distance of Centres of star and planet, and the Position Angle of the centre of the planet reckoned with respect to the star as origin. These quantities s and p_0 , in columns 20 and 21 respectively, have been computed by the formulæ—

$$\begin{aligned}\sin \frac{1}{2} s \cdot \sin p_0 &= \cos \frac{1}{2} (\delta' + \tilde{\epsilon}) \sin \frac{1}{2} (a' - a) \\ \sin \frac{1}{2} s \cdot \cos p_0 &= \sin \frac{1}{2} (\tilde{\epsilon}' - \tilde{\epsilon}) \cos \frac{1}{2} (a' - a),\end{aligned}$$

where

α and δ are respectively the R.A. and Dec. of the star from column 19.

and

α' and δ' are the apparent R.A. and Dec. of the planet computed as follows.

$$\begin{aligned}\alpha' &= \text{R.A. (from column 15).} \\ &+ \text{Tab. Parallax in R.A. (from column 17).} \\ &+ \Delta\alpha \text{ (from column 19).} \\ \delta' &= \text{Dec. (from column 16).} \\ &+ \text{Tab. Parallax in Dec. (from column 18).} \\ &+ \Delta\delta \text{ (from column 19).}\end{aligned}$$

On October 3, when position angles were measured for determining the parallax, it was necessary to compute not p_0 (the mean of the angles at the star and planet), but to find the tabular quantity for the angle actually measured, viz. the position angle at the middle point between the star and planet. In these cases a correction was applied to the values of p_0 , found by the above formulæ, viz.

$$-\frac{1}{16} s^2 \sin 1'' \sin 2p (1 + 2 \tan^2 \delta).$$

All the computations described in this Chapter have been performed in duplicate; the separate computations were then carefully examined, and finally the independent results were compared; I hardly think it possible that any arithmetical error remains undetected.

CHAPTER XIII.

COMPARISON OF OBSERVED AND TABULAR QUANTITIES.

The quantities of column 20, Chapter XII., give the tabular distance between the star of comparison and the *true* centre of the planet. In reducing the instrumental distance, therefore, a correction must be applied on account of phase, every measure of distance being affected with half the defect of illumination at the part of the limb whose position angle with respect to the

centre of the planet is the same as, or 180° different from, the position angle of the centre of the planet with respect to the star of comparison.

If Q = the position angle of greatest defect of illumination*

M = the areocentric angle between the Earth and the Sun,

and $\tan m = \tan M \cos (p_0 - Q)$.

Then the correction of the observed distance for defective illumination, which we shall call P , is given by the formula

$$P = (\text{semidiameter of Mars}) \times \sin^2 \frac{1}{2} m. \dagger$$

the sign of which correction is + when the numerical difference between p_0 and Q is less than 90° .

To convert the measured distances into arc, a mean value of the screw (derived from a separate discussion) has been adopted, viz.

$$1 \text{ revolution} = 25''.72721.$$

and to the resulting distances is added the value of P . The results so found are given in column 6 of the following Table II.

In making the comparisons ($O-C$), contained in Table II., the results of each of the four observations which compose each group have been separately computed, in order more perfectly to detect possible errors of scale or screw readings or of computation,—errors which, though perhaps obvious enough in separate observations, might be passed over in the mean of the group.

Thus in the example quoted in Chapter XII. the separate results are—

* The values of Q have been taken from Mr. MARTH's *Ephemeris for Physical Observations of Mars*, 1877. *Monthly Notices*, Vol. XXXVII. pp. 302-303.

† The separate values of P are given for each group in the O.M. Chap. XIII. Table I.

	Number.	O.	C.	O - C.
Group 43*	171	4364''85	4364''53	+ 0''32 + Δθ†
	172	65'23	65'16	+ 07 - Δθ
	173	65'52	65'55	- 03 - Δθ
	174	66'71	66'41	+ 30 + Δθ
Mean ...		4365'58	4365'41	+ 0'17

In the above case the observations were made by Method A, in which the image of the star is placed in the centre of the planet.

In the *Monthly Notices*, vol. xxxix. p. 61, is given an example of a similar comparison when Method B is employed.‡

As compared with computing the mean results of each group only, the labour of this detailed comparison is exactly fourfold ; yet I could not satisfy myself that every possible error had been detected until the work of comparison, as above described, had been performed.

The last three columns of the following Table contain the factors for R.A., Dec., and Parallax respectively.

Where

$$f' = \sin p_0 \cos \delta_0$$

$$f'' = \cos p_0$$

$$f''' = \left(\frac{\text{Parallax in R.A.}}{100} \right) f' + \left(\frac{\text{Parallax in Dec.}}{100} \right) f''.$$

the last factor, therefore, expresses the effect on the tabular distance of increasing the tabular parallax by *one per cent.*

* The correction of the observed distance for phase is -0''62 in this group.

† Δθ is the outstanding index error.

‡ In the final results printed in this chapter, the value of the Measured Distance differs +0''03, and the Tabular Distance differs -0''52 from the corresponding example in the *Monthly Notices*. The former difference is due to an increase of 0''00021 in the final adopted screw value, and the latter to the application of the final corrections to the places of the stars of comparison.

TABLE II.

Group.	Mean Zenith Distance of Group.	Mode of Obser- vation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of <i>Mars</i> from Star.	O-C.	Coefficients of		
								$\Delta\alpha$ "	$\Delta\delta$ "	κ "
Evening Observations, July 31.										
42	65°	A	<i>q</i>	$\overset{r}{270}^{\circ}6197$	6962''13	6961''32	+0''81	+183	+983	+047
43	60	A	<i>n</i>	169°7113	4365'58	4365'41	+0'17	+976	+136	+172
44	53	A	<i>t</i>	163°0152	4194'54	4194'48	+0'06	-902	-404	-147
45	49	A	<i>t</i>	162°9537	4192'96	4192'42	+0'54	-902	-404	-137
46	42	A	<i>n</i>	170°1143	4375'95	4375'78	+0'17	+976	+135	+134
47	39	A	<i>q</i>	270°5131	6959'39	6958'97	+0'42	+185	+982	+025
Morning Observations, July 31.										
48	17°	A	<i>q</i>	$\overset{r}{270}^{\circ}3453$	6955''07	6954''54	+0''53	+189	+981	-016
49	22	A	<i>n</i>	171°2504	4405'19	4404'29	+0'90	+976	+131	-071
50	25	A	<i>t</i>	161°8980	4165'80	4165'05	+0'75	-899	-410	+080
51	29	A	<i>t</i>	161°8423	4164'37	4163'56	+0'81	-899	-410	+089
52	35	A	<i>n</i>	171°4444	4410'18	4410'55	-0'37	+976	+131	-112
53	39	A	<i>q</i>	270°3405	6954'94	6953'84	+1'10	+191	+981	-023
Evening Observations, August 5.										
55	70°	A	<i>q</i>	$\overset{r}{261}^{\circ}5262$	6728''16	6727''90	+0''26	+317	+946	+078
56	67	A	<i>t</i>	142°6520	3670'50	3670'00	+0'50	-796	-590	-161
57	62	A	<i>n</i>	202°2590	5203'04	5203'06	-0'02	+985	+023	+183
58	59	A	<i>n</i>	202°2399	5202'55	5203'03	-0'48	+985	+022	+178
59	55	A		142°7698	3673'54	3672'99	+0'55	-796	-591	-141
60	51	A	<i>q</i>	261°2205	6720'29	6719'70	+0'59	+318	+946	+057
Morning Observations, August 5.										
61	19°	A	<i>q</i>	$\overset{r}{260}^{\circ}0314$	6689''70	6689''75	-0''05	+317	+946	-026
62	23	A	<i>t</i>	143°8902	3702'36	3702'39	-0'03	-793	-595	+070
63	37	A	<i>n</i>	201°5985	5186'06	5186'35	-0'29	+985	+015	-124
64	41	A	<i>n</i>	201°5965	5186'01	5185'69	+0'32	+985	+015	-135

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	κ f'''
Morning Observations, August 10.										
66	42°	A	n	189° 9459	4886".42	4886".09	+0".33	+ '972	— '163	— '141
67	45	A	t	176° 7494	4547".59	4547".20	+0".39	— '725	— '678	+ '110
68*	49	A	q*	222° 8689	5733".67	5733".60	+0".07	+ '306	+ '950	— '046
68*	49	A	q*	222° 8689	5733".67	5733".60	+0".07	+ '306	+ '950	— '046
69	51	A	t	176° 9998	4554".03	4553".46	+0".57	— '726	— '678	+ '120
70	55	A	n	189° 5920	4877".31	4877".13	+0".18	+ '972	— '164	— '174
Evening Observations, August 12.										
71	66°	A	q	205° 3542	5283".10	5283".28	—0".18	+ '268	+ '962	+ '069
72	63	A	t	196° 5472	5056".90	5056".78	+0".12	— '720	— '683	— '150
73	59	A	n	179° 3043	4612".69	4612".56	+0".13	+ '952	— '253	+ '178
74	56	A	n	179° 2286	4610".74	4610".63	+0".11	+ '952	— '254	+ '174
75	54	A	t	196° 9849	5068".16	5067".75	+0".41	— '720	— '682	— '133
76	50	A	q	204° 7667	5267".98	5268".10	—0".12	+ '266	+ '963	+ '051
Morning Observations, August 13.										
78	32°	B	q	190° 0794	4890".15	4890".46	—0".31	+ '208	+ '977	— '028
80	39	B	n	167° 4975	4308".99	4309".69	—0".70	+ '925	— '342	— '130
81	55	B	n	166° 9760	4295".57	4296".23	—0".66	+ '925	— '345	— '170
82	58	B	t	217° 3856	5592".99	5592".80	+0".19	— '728	— '674	+ '131
Evening Observations, August 14.*										
88	65°	B	t	226° 8738	5837".08	5837".43	—0".35	— '728	— '673	— '156
89	60	B	n	162° 3867	4177".53	4177".56	—0".03	+ '904	— '394	+ '172
92*	51	B	q*	181° 1366	4660".10	4661".19	—1".09	+ '170	+ '985	+ '036
90	57	B	n	162° 2933	4175".13	4175".27	—0".14	+ '904	— '395	+ '167
91	55	B		227° 4329	5851".47	5851".18	+0".29	— '728	— '673	— '138
92*	51	B	q*	181° 1366	4660".10	4661".19	—1".09	+ '170	+ '985	+ '036

* The observations of this evening are very unsatisfactory and discordant *inter se*, perhaps it is safest not to use them.

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	κ f'''
Morning Observations, August 14.										
93	33°	B	q	$\overset{r}{178^{\circ}1248}$	5482''62	4583''11	—0''49	+ '148	+ '989	+ '021
94	41	B	t	232'9867	5594'34	5994'74	—0'40	— '733	— '668	+ '109
95	47	B	n	158'4683	4076'73	4077'01	—0'28	+ '892	— '423	— '146
Evening Observations, August 15.										
97	58°	A	q	$\overset{r}{169^{\circ}5685}$	4362''50	4363''21	—0''71	+ '098	+ '995	+ '028
98	56	A	s	179'2428	4611'59	4610'95	+0'64	— '588	— '802	— '116
99	52	A	n	153'2803	3943'27	3943'18	+0'09	+ '861	— '484	+ '151
100	49	A	n	153'1692	3940'41	3940'89	—0'48	+ '860	— '485	+ '146
101	47	A	s	179'7184	4623'82	4623'03	+0'79	— '589	— '801	— '100
102	44	A	q	169'0734	4349'76	4350'30	—0'54	+ '094	+ '995	+ '017
Morning Observations, August 15.										
103	30°	A	q	$\overset{r}{166^{\circ}5442}$	4284''51	4285''65	—1''15	+ '072	+ '997	— '012
104	34	A	s	184'4532	4745'64	4744'78	+0'86	— '600	— '793	+ '078
105	39	A	n	149'9964	3858'81	3858'84	—0'03	+ '844	— '513	— '120
106	47	A	n	149'7334	3852'04	3852'14	—0'10	+ '843	— '515	— '140
Evening Observations, August 16.										
108	62°	B	q	$\overset{r}{158^{\circ}5466}$	4078''95	4079''72	—0''77	+ '006	+ '999	+ '014
109	54	B	s	197'5383	5082'28	5082'47	—0'19	— '617	— '780	— '116
110	50	B	n	144'8510	3726'46	3726'00	+0'46	+ '799	— '584	+ '136
111	47	B	n	144'7680	3724'32	3724'28	+0'04	+ '798	— '585	+ '132
112	45	B	s	198'0001	5094'16	5094'54	—0'38	— '617	— '780	— '101
113	43	B	q	157'8947	4062'18	4063'45	—1'27	— '002	+ '999	+ '001
Morning Observations, August 16.										
114	53°	B	q	$\overset{r}{155^{\circ}2371}$	3993''81	3993''84	—0''04	— '039	+ '999	+ '014
115	56	B	s	204'3549	5257'65	5257'13	+0'52	— '628	— '770	+ '112
116	60	B	n	141'3717	3636'97	3636'58	+0'39	+ '771	— '621	— '158

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f'''
Evening Observations, August 19.										
123	66°	B	m	^r 210°1728	5407''09	5407''79	—0''70	+ '433	+ '896	+ '104
124	63	B	n	131°3127	3378'28	3377'47	+ 0'81	+ '447	— '891	+ '080
125	61	B	r	174°0686	4478'48	4479'74	—1'26	— '914	+ '366	— '179
126	57	B	r	174°2517	4483'20	4484'63	—1'43	— '915	+ '365	— '173
127	54	B	n	131°3059	3378'11	3377'29	+ 0'82	+ '442	— '893	+ '076
128	51	B	m	209°2645	5383'72	5384'00	—0'28	+ '430	+ '899	+ '082
Morning Observations, August 19.										
130	43°	B	m	^r 203°4745	5234''77	5234''90	—0''13	+ '408	+ '910	— '064
131	47	B	n	131°0383	3371'24	3370'39	+ 0'85	+ '385	— '920	— '067
132	50	B	r	180°6771	4648'50	4649'13	—0'63	— '926	+ '333	+ '167
Evening Observations, August 20.										
134	61°	B	n	^r 133°6705	3438''96	3438''68	+ 0''28	+ '271	— '961	+ '044
135	60	B	r	191°8613	4936'24	4937'30	—1'06	— '945	+ '272	— '184
136	56	B	λ	256°6594	6602'93	6601'92	+ 1'01	+ '931	+ '319	+ '182
137	53	B	λ	256°3884	6595'95	6595'76	+ 0'19	+ '931	+ '319	+ '175
138	51	B	r	192°3956	4949'98	4951'39	—1'41	— '946	+ '270	— '168
139	48	B	n	133°8813	3444'38	3443'46	+ 0'92	+ '264	— '963	+ '042
Morning Observations, August 21.										
151*	26°	B	n	^r 143°2298	3684''90	3684''08	+ 0''82	+ '039	— '999	+ '004
152	28	B	r	219°2011	5639'60	5640'44	—0'84	— '967	+ '171	+ '107
153	33	B	λ	222°1490	5715'10	5714'38	+ 0'72	+ '936	+ '303	— '118
154	62	B	λ	219°5899	5649'27	5648'99	+ 0'28	+ '936	+ '302	— '188
155	65	B	r	221°9856	5711'25	5712'31	—1'06	— '968	+ '164	+ '208
151*	26	B	n	143°2298	3684'90	3684'08	+ 0'82	+ '039	— '999	+ '004

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	μ f'''
Evening Observations, August 23.*										
160	70°	B	n	168°1021	4324".82	4324".45	+0".37	—'225	—'973	—'067
161	58	B	r	261°3936	6725".08	6727".04	—1".96	—'979	+ '064	—'190
162	51	B	l	240°7442	6193".54	6193".42	+0".12	+ '980	+ '052	+ '182
163	48	B	l	240°3934	6184".51	6185".30	—0".79	+ '980	+ '051	+ '173
164	47	B	r	262°3338	6749".26	6750".39	—1".13	—'979	+ '062	—'164
165	42	B	r	262°6956	6758".57	6759".64	—1".07	—'980	+ '061	—'153
166	38	B	l	239°4838	6161".11	6160".64	+0".47	+ '980	+ '050	+ '142
Evening Observations, August 24.										
171	66°	B	n	188°1727	4841".18	4840".91	+0".27	—'348	—'935	—'090
172	53	B	q	196°4480	5054".18	5054".28	—0".10	—'923	+ '338	—'171
173	48	B	l	211°9852	5453".66	5453".51	+0".15	+ '981	+ '001	+ '172
174	44	B	l	211°6265	5444".43	5444".51	—0".08	+ '981	'000	+ '162
175	42	B	q	197°2348	5074".42	5074".67	—0".25	—'924	+ '335	—'145
176	39	B	n	190°0157	4888".59	4888".07	+0".52	—'359	—'931	—'050
Morning Observations, August 24.										
177	30°	B	q	202°9581	5221".67	5221".94	—0".27	—'931	+ '313	+ '108
178	36	B	l	204°0319	5249".05	5248".73	+0".32	+ '981	—'013	—'134
179	40	B	n	195°3728	5026".42	5026".40	+0".02	—'387	—'919	+ '061
180	44	B	l	203°2653	5229".33	5229".42	—0".09	+ '981	—'014	—'158
181	45	B	q	204°1417	5252".12	5252".94	—0".82	—'933	+ '309	+ '157
182	53	B	q	204°7788	5268".51	5268".62	—0".11	—'934	+ '307	+ '178
183	57	B	l	202°0369	5197".72	5197".47	+0".25	+ '981	—'017	—'193
184	59	B	n	196°6690	5059".76	5058".82	+0".94	—'393	—'916	+ '071

* The observations of this evening were not satisfactory. In the original note-book the three first observations are condemned as useless. Besides this, there are no corresponding morning observations, whilst on the following day excellent corresponding observations were obtained both evening and morning. The introduction of the observations of August 23 into the final discussion might do harm, but could be of no use. They are therefore rejected.

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Stars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f'''
Evening Observations, August 26.*										
185	69°	B	n	$\overset{r}{236.6335}$	6087".93	6087".37	+0".56	—'.525	—'.845	—'.129
186	67	B	q	246.8293	6350.34	6349.64	+0.70	—'.965	+'.173	—'.206
187	63	B	l	155.4715	3999.75	4000.32	—0.57	+'.970	—'.151	+'.203
188	60	B	l	155.2642	3994.55	3994.80	—0.25	+'.969	—'.152	+'.198
189	53	B	q	248.0682	6382.20	6382.29	—0.09	—'.965	+'.170	—'.177
190	41	B	q	249.1603	6410.30	6410.70	—0.40	—'.966	+'.167	—'.146
191	37	B	n	239.3850	6158.74	6157.65	+1.09	—'.532	—'.841	—'.070
Evening Observations, August 27.*										
192	72°	B	n	$\overset{r}{264.2907}$	6799".49	6798".69	+0".80	—'.586	—'.803	—'.145
193	64	B	h	266.2451	6849.69	6849.65	+0.04	+'.855	+'.486	+'.190
194	60	B	l	127.1214	3270.40	3270.24	+0.16	+'.942	—'.282	+'.191
195	58	B	m	156.0151	4013.90	4014.79	—0.89	—'.786	+'.596	—'.152
196	52	B	λ	47.5284	1222.68	1222.16	+0.52	+'.979	—'.049	+'.184
197	46	B	m	156.7371	4032.47	4032.92	—0.45	—'.791	+'.590	—'.133
198	42	B	l	125.5963	3231.17	3230.97	+0.20	+'.939	—'.290	+'.151
199	38	B	h	263.4574	6777.96	6778.56	—0.60	+'.854	+'.487	+'.126
200	35	B	n	267.5908	6884.39	6883.40	+0.99	—'.593	—'.798	—'.074
Evening Observations, August 29.										
203	66°	A	i	$\overset{r}{225.1560}$	5792".61	5791".57	+1".04	+'.535	—'.839	+'.105
204	62	A	m	202.6560	5213.83	5214.01	—0.18	—'.920	+'.340	—'.189
205	60	A	m	202.8668	5219.25	5219.57	—0.32	—'.921	+'.339	—'.185
206	54	A	f	271.5505	6986.18	6986.93	—0.75	+'.970	—'.143	+'.188
207	51	B	h	195.8983	5039.89	5040.82	—0.93	+'.827	+'.534	+'.155
208	48	B	λ	32.5907	838.48	837.53	+0.95	—'.567	—'.816	—'.100
209	45	B	m	204.1674	5252.72	5252.80	—0.08	—'.922	+'.335	—'.153

* The observations of August 26 and 27 are of no use for the parallax determination, as they are without corresponding morning observations. They may, however, be useful in the determination of the law of variation of the tabular error of the planet.

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f'''
Morning Observations, August 29.										
210	25°	B	λ	$\overset{r}{39^{\circ}51'27''}$	1016''57	1015''93	+0''64	−'664	−'735	+ '075
211	28	B	h	187'2426	4817'20	4817'18	+0'02	+ '821	+ '545	−'095
212	30	B	m	211'0186	5428'98	5430'38	−1'40	−'930	+ '311	+ '109
213	41	A	f	261'7992	6735'31	6735'70	−0'39	+ '967	−'161	−'146
214	42	A	m	212'2597	5460'90	5460'68	+0'22	−'931	+ '308	+ '151
215	45	A	m	212'5241	5467'70	5467'39	+0'31	−'932	+ '307	+ '159
216	49	A	h	184'9112	4757'22	4757'50	−0'28	+ '818	+ '548	−'146
Evening Observations, August 31.*										
212 bis	50°	A	f	$\overset{r}{208^{\circ}71'82''}$	5369''70	5370''20	−0''50	+ '935	−'300	+ '171
213 bis	44	A	m	265'3003	6825'49	6825'25	+0'24	−'964	+ '168	−'157
214 bis	41	A	m	265'5956	6833'08	6832'50	+0'58	−'964	+ '168	−'149
215 bis	36	A	f	207'4684	5337'54	5338'11	−0'57	+ '934	−'304	+ '137
Morning Observations, August 31.										
216 bis	25°	A	f	$\overset{r}{201^{\circ}46'16''}$	5183''00	5182''75	+0''25	+ '927	−'324	−'087
217 bis	29	A	m	273'0848	7025'75	7025'80	−0'05	−'966	+ '155	+ '117
218	35	A	m	273'6987	7041'55	7041'45	+0'10	−'966	+ '154	+ '135
219	43	A	f	199'7936	5140'09	5139'21	+0'88	+ '926	−'329	−'145
Evening Observations, September 2.										
221	54°	B	g	$\overset{r}{111^{\circ}15'43''}$	2859''68	2858''94	+0''74	+ '920	+ '338	+ '179
222	52	B	λ	174'9590	4501'22	4501'72	−0'50	−'890	−'416	−'164
223	48	B	λ	175'3389	4511'00	4510'88	+0'12	−'891	−'416	−'158
224	44	B	g	109'9966	2829'90	2829'42	+0'48	+ '920	+ '338	+ '153
225	41	B	g	109'5917	2819'47	2819'25	+0'22	+ '920	+ '338	+ '143
226	37	B	λ	176'7331	4546'86	4546'81	+0'05	−'891	−'415	−'124
227	32	A	k	124'1606	3194'33	3194'65	−0'32	−'190	+ '981	−'033
228	29	A	i	237'6066	6112'95	6111'56	+1'39	−'093	−'996	−'001

* Note (from original note-book), 'The definition unsatisfactory during groups 212 bis, 213 bis, 214 bis, 215 bis—allow $\frac{1}{2}$ weight.'

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O-C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	κ f'''
Evening Observations, September 4.										
229	69°	B	f	$\overset{r}{120}^{\circ}31'73$	3095''41	3094''27	+1''14	+·457	-·884	+·089
230	65	B	k	137°8426	3546'33	3546'75	-0'42	-·662	+·735	-·134
231	61	B	g	35°9388	924'61	924'55	+0'06	+·873	+·450	+·186
232	58	B	g	35°6174	916'34	915'08	+1'26	+·872	+·452	+·179
233	55	B	g	35°2843	907'78	906'74	+1'04	+·871	+·453	+·173
234	52	B	g	34°9262	898'56	898'63	-0'07	+·871	+·455	+·166
235	49	A	k	138°6652	3567'49	3568'26	-0'77	-·671	+·727	-·121
236	46	A	f	119°7635	3081'16	3080'33	+0'83	+·438	-·895	+·076
Evening Observations, September 5.										
239	68°	A	f	$\overset{r}{118}^{\circ}2036$	3041''03	3039''88	+1''15	+·158	-·987	+·021
240	65	A	h	87°8163	2259'28	2260'30	-1'02	-·929	+·258	-·198
241	59	B	g	7°7652	199'80	201'00	-1'20	-·644	+·752	-·127
242	56	B	g	7°9145	203'64	204'87	-1'23	-·671	+·727	-·129
243	52	B	g	8°1910	210'75	211'77	-1'02	-·711	+·686	-·132
244	49	B	g	8°3290	214'31	215'33	-1'02	-·732	+·666	-·132
245	47	B	g	8°5046	218'82	220'07	-1'25	-·749	+·642	-·132
246	44	B	g	8°7026	223'92	224'92	-1'00	-·768	+·619	-·130
Morning Observations, September 5.*										
247	11°	B	g	$\overset{r}{13}^{\circ}8851$	357''23	357''90	-0''67	-·940	+·275	+·036
248	14	B	g	14°1550	364'18	364'92	-0'74	-·943	+·265	+·046
249	17	B	g	14°5710	374'88	375'40	-0'52	-·946	+·250	+·062
250	20	B	g	14°8357	381'69	383'01	-1'32	-·949	+·240	+·075
251	22	B	g	15°1125	388'81	390'12	-1'31	-·951	+·231	+·082

* For the observations of morning, September 5, the scale value is determined by a measurement of the standard distance *e-g*. See triangulation of comparison stars, measure 253.

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O - C.	Coefficients of		
								$\Delta\alpha$ <i>f'</i>	$\Delta\delta$ <i>f''</i>	κ <i>f'''</i>
Morning Observations, September 5—(continued).										
252	25°	B	<i>g</i>	$\overset{r}{15^{\circ}43'00''}$	396".98	397".94	-0".96	-'.953	+'.222	+'.092
254	43	B	<i>g</i>	17°39'04	447".41	448".53	-1".12	-'.962	+'.170	+'.155
255	45	B	<i>g</i>	17°66'41	454".46	454".84	-0".38	-'.963	+'.165	+'.159
257	56	B	<i>g</i>	18°88'18	485".79	485".93	-0".14	-'.965	+'.155	+'.174
258	59	B	<i>g</i>	19°22'99	494".73	495".44	-0".71	-'.968	+'.140	+'.192
259	62	B	<i>g</i>	19°56'69	503".39	503".96	-0".57	-'.969	+'.126	+'.207
Evening Observations, September 6.										
260	69°	A	<i>f</i>	$\overset{r}{127^{\circ}73'58''}$	3285".27	3285".43	+0".84	-.139	-.990	-.047
261	56	B	<i>g</i>	42°78'62	1100".77	1101".89	-1".12	-.973	-.094	-.192
262	53	B	<i>g</i>	43°12'69	1109".54	1109".28	+0".26	-.972	-.095	-.187
263	49	B	<i>e</i>	230°76'57	5936".96	5936".15	+0".81	+'.969	+'.125	+'.177
264	46	B	<i>e</i>	230°41'25	5927".87	5927".74	+0".13	+'.969	+'.125	+'.170
265	45	B	<i>h</i>	124°13'60	3193".68	3194".11	-0".43	-.968	+'.134	-.162
266	42	B	<i>g</i>	44°42'34	1142".89	1143".06	-0".17	-.971	-.100	-.155
267	40	B	<i>g</i>	44°68'31	1149".57	1149".67	-0".10	-.971	-.100	-.156
268	37	B	<i>f</i>	129°24'81	3325".17	3324".61	+0".56	-.164	-.986	-.017
269	35	B	<i>f</i>	129°33'70	3327".46	3327".15	+0".31	-.166	-.985	-.015
Morning Observations, September 6.										
270	24°	B	<i>g</i>	$\overset{r}{52^{\circ}37'13''}$	1347".36	1348".22	-0".86	-.970	-.119	+'.094
271	35	B	<i>e</i>	220°55'89	5674".37	5673".44	+0".93	+'.970	+'.121	-.131
272	39	B	<i>e</i>	220°12'80	5663".28	5662".15	+1".13	+'.970	+'.120	-.143
273	45	B	<i>h</i>	134°45'69	3459".21	3460".07	-0".86	-.972	+'.106	+'.166
274	53	B	<i>g</i>	55°9'255	1438".81	1439".73	-0".92	-.969	-.124	+'.185

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f'''
Evening Observations, September 7.										
276	68°	A	<i>h</i>	$\overset{r}{157}^{\circ}7351$	4058''08	4058''06	+ 0''02	— '976	+ '050	— '214
277	61	A	<i>e</i>	194'1469	4994'86	4994'61	+ 0'25	+ '972	+ '102	+ '202
278	44	B	<i>e</i>	192'3826	4949'46	4948'94	+ 0'52	+ '972	+ '101	+ '167
279	44	B	<i>g</i>	82'2329	2115'62	2116'17	— 0'55	— '963	— '166	— '158
280	42	B	<i>g</i>	82'5217	2123'05	2123'17	— 0'12	— '964	— '167	— '151
281	39	B	<i>f</i>	148'7357	3826'54	3826'91	— 0'37	— '387	— '919	— '052
282	37	B	<i>k</i>	219'6499	5650'98	5651'33	— 0'35	— '923	+ '324	— '134
283	32	B	γ	252'2271	6489'10	6489'66	— 0'56	+ '790	+ '586	+ '097
Morning Observations, September 7.										
286	26°	B	<i>e</i>	$\overset{r}{183}^{\circ}7034$	4726''18	4726''03	+ 0''15	+ 972	+ '096	— '101
287	29	B	<i>g</i>	91'0446	2342'33	2342'71	— 0'38	— '962	— '171	+ '114
288*	56	B	<i>g</i>	94'3118	2426'38	2425'98	+ 0'40	— '962	— '171	+ '189
Evening Observations, September 8.										
290	66°	B	<i>h</i>	$\overset{r}{194}^{\circ}6614$	5008''10	5008''51	— 0''41	— '977	— '005	— '214
291	53	B	<i>e</i>	155'5727	4002'45	4003'03	— 0'58	+ '975	+ '070	+ '197
292	52	B	<i>g</i>	119'3483	3070'50	3070'19	+ 0'31	— '959	— '187	— '178
293	49	B	<i>g</i>	119'6011	3077'00	3076'56	+ 0'44	— '959	— '187	— '173
294	45	B	<i>e</i>	154'6839	3979'58	3980'15	— 0'57	+ '975	+ '069	+ '167
295	44	B	<i>h</i>	197'2454	5074'57	5074'28	+ 0'29	— '977	— '008	— '158
296	39	B	<i>e</i>	153'9932	3961'81	3961'82	— 0'01	+ '975	+ '068	+ '147

* There was a long delay by cloud between observations 287 and 288.

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of <i>Mars</i> from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ <i>f'</i>	$\Delta\delta$ <i>f''</i>	<i>n</i> <i>f'''</i>
Morning Observations, September 8.										
297	18 ²	B	<i>d</i>	^r 269·8912	6943 ^u 54	6942 ^u 54	+ 1 ^u 00	+ ·721	— ·677	— ·039
298	22	B	<i>k</i>	258·8565	6659·66	6660·72	— 1·06	— ·949	+ ·236	+ ·083
299	27	B	γ	211·4424	5439·84	5440·25	— 0·41	+ ·740	+ ·651	— ·085
300	30	B	<i>f</i>	180·0728	4632·75	4632·90	— 0·15	— ·565	— ·817	+ ·077
301	35	B	<i>e</i>	145·1874	3735·27	3735·23	+ 0·04	+ ·975	+ ·060	— ·130
302	36	B	<i>g</i>	129·9153	3342·36	3342·14	+ 0·22	— ·959	— ·188	+ ·138
301	35	B	<i>e</i>	145·1874	3735·27	3735·23	+ 0·04	+ ·975	+ ·060	— ·130
303	39	B	<i>h</i>	207·0050	5325·66	5325·20	+ 0·46	— ·976	— ·017	+ ·148
305*	58	B'	<i>e</i>	142·4038	3663·65	3662·89	+ 0·76	+ ·975	+ ·058	— ·196
306*	61	B'	<i>h</i>	209·5490	5391·11	5391·01	+ 0·10	— ·977	— ·019	+ ·205
Evening Observations, September 9.										
307	50 ⁰	B	<i>e</i>	^r 118·0848	3038 ^u 00	3037 ^u 72	+ 0 ^u 28	+ ·977	+ ·021	+ ·177
308	47	B	<i>g</i>	157·4791	4051·50	4052·13	— 0·63	— ·958	— ·195	— ·165
Morning Observations, September 9.										
309	26 ⁰	B	<i>d</i>	^r 249·7341	6424 ^u 95	6423 ^u 64	+ 1 ^u 31	+ ·630	— ·765	— ·054
310	30	B	θ	248·7503	6399·67	6400·03	— 0·36	— ·428	+ ·898	+ ·041
311	34	B	<i>g</i>	167·1268	4299·70	4299·51	+ 0·19	— ·958	— ·194	+ ·128
312	43	B	<i>e</i>	107·3064	2760·69	2760·14	+ 0·55	+ ·976	+ ·004	— ·154
313	48	B	<i>e</i>	106·6462	2743·71	2743·18	+ 0·53	+ ·976	+ ·003	— ·172
314	53	B	<i>g</i>	169·4350	4359·08	4359·21	— 0·13	— ·958	— ·194	+ ·183
315	57	B	<i>g</i>	169·8269	4369·16	4369·51	— 0·35	— ·958	— ·194	+ ·194
316	61	B	<i>e</i>	105·2307	2707·30	2706·79	+ 0·51	+ ·976	+ ·001	— ·201
317	63	B	<i>g</i>	170·4888	4386·19	4386·51	— 0·32	— ·958	— ·194	+ ·201
318	67	B	<i>e</i>	104·5795	2690·54	2690·01	+ 0·53	+ ·976	·000	— ·212

* Observations 305 and 306 were made without the reversing prism.

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								Δa f'	$\Delta \delta$ f''	n f'''
Evening Observations, September 10.										
319	51°	B	e	$\overset{r}{81}^{\circ}9825$	2109''19	2109''03	+0''16	+·974	—·063	+·181
320	49	B	g	194·4953	5003·81	5003·53	+0·28	—·957	—·197	—·169
321	47	B	g	194·7827	5011·21	5010·28	+0·93	—·957	—·197	—·162
322	42	B	e	80·9784	2083·35	2083·44	—0·09	+·974	—·066	+·157
323	40	B	θ	257·1645	6616·13	6616·50	—0·37	—·521	+·845	—·084
325	33	B	d	237·0212	6097·89	6097·46	+0·43	+·542	—·832	+·078
326	32	B	f	234·4646	6032·09	6031·83	+0·26	—·709	—·689	—·078
327	27	B	γ	154·6583	3978·96	3980·09	—1·13	+·591	+·795	+·052
Morning Observations, September 10.										
329	39°	B	e	$\overset{r}{71}^{\circ}7506$	1845''94	1845''32	+0''62	+·971	—·098	—·143
330	42	B	g	205·1521	5277·98	5278·30	—0·32	—·958	—·196	+·153
331	45	B	g	205·5913	5289·28	5289·02	+0·26	—·958	—·196	+·163
332	51	B	e	70·4366	1812·06	1811·64	+0·42	+·971	—·103	—·177
333	54	B	e	70·1168	1803·91	1802·95	+0·96	+·971	—·104	—·185
334	56	B	g	206·8043	5320·48	5320·29	+0·19	—·958	—·196	+·188
335	59	B	g	207·1026	5328·16	5327·76	+0·40	—·958	—·195	+·193
336	63	B	e	69·1183	1778·21	1777·81	+0·40	+·970	—·108	—·204
Evening Observations, September 11.										
338	65°	B	g	$\overset{r}{229}^{\circ}3764$	5901''19	5900''88	+0''31	—·958	—·196	—·204
339	62	B	g	229·6657	5908·64	5908·15	+0·49	—·958	—·196	—·201
340	58	B	μ	151·4396	3896·14	3896·47	—0·33	+·942	+·263	+·191
341	55	B	μ	151·1155	3887·80	3888·57	—0·77	+·942	+·263	+·184
342	53	B	g	230·7450	5936·41	5936·12	+0·29	—·958	—·196	—·178
343	47	B	e	47·0415	1210·25	1210·27	—0·02	+·942	—·262	+·165
Evening Observations, September 15.										
344	51°	A	e	$\overset{r}{226}^{\circ}2601$	5821''12	5820''89	+0''23	+·757	+·628	+·137
345	48	A	e	97·9796	2520·67	2521·66	—0·99	—·915	—·347	—·155

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f''
Evening Observations, September 16.										
346	54°	B	μ	$\overset{r}{27}7805$	714''71	715''61	—0''90	—'833	+ '511	—'154
347	41	B	μ	28'8498	742'22	743'52	—1'30	—'852	+ '486	—'131
Evening Observations, September 17.										
350	63°	A	c	$\overset{r}{177}1036$	4556''40	4555''62	+0''78	+ '626	+ '766	+ '136
351	61	B	e	159'3714	4100'09	4101'27	—1'18	—'943	—'254	—'189
352	56	B	β	117'3984	3020'28	3019'83	+0'45	+ '098	—'995	+ '016
354	39	B	μ	57'6362	1482'78	1484'23	—1'45	—'957	+ '192	—'140
355	33	A	β	117'4214	3020'87	3020'30	+0'57	+ '079	—'997	+ '021
356	29	A	c	174'6846	4494'16	4493'68	+0'48	+ '616	+ '775	+ '059
357	24	A	e	162'9239	4191'48	4192'79	—1'31	—'945	—'251	—'083
Morning Observations, September 17.										
358	57°	A	β	$\overset{r}{117}6132$	3025''80	3024''97	+0''83	+ '006	—'977	—'005
359	68	A	e	171'8181	4420'32	4421'45	—1'13	—'948	—'241	+ '198
Evening Observations, September 19.*										
361	49°	A	b	$\overset{r}{240}9484$	6199''03	6197''30	+1''73	+ '974	+ '032	+ '171
362	44	B	e	219'4446	5645'58	5647'11	—1'53	—'955	—'202	—'147
363	40	A	c	141'8118	3648'54	3648'63	—0'09	+ '361	+ '929	+ '046
364	38	B	e	220'0080	5660'08	5661'18	—1'10	—'955	—'202	—'129
365	33	A	b	239'3595	6158'15	6157'97	+0'18	+ '974	+ '032	+ '120

* The definition was not satisfactory during the observations of September 19, the observations are only entitled to $\frac{1}{2}$ weight. In the final discussion this has been done by taking the mean of 361 and 365 as one equation, and the mean of 362 and 364 as one equation. The parallax factor of 363 is so small that no harm is done by giving that equation weight = 1.

TABLE II.—continued.

Group.	Mean Zenith Distance of Group.	Mode of Observ- ation.	Star of Com- parison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								$\Delta\alpha$ f'	$\Delta\delta$ f''	n f'''
Morning Observations, September 19.										
366	24°	A	c	^r 139°5798	3591''11	3590''94	+ 0''17	+ '327	+ '942	— '042
367	32	A	b	233°3956	6004'72	6002'91	+ 1'81	+ '974	+ '032	— '110
368	37	B	e	226°8987	5837'35	5838'14	— 0'79	— '957	— '197	+ '135
369*	43	B'	e	227°4624	5851'86	5852'07	— 0'21	— '957	— '196	+ '151
370*	60	B'	b	230°8181	5938'41	5937'21	+ 1'20	+ '974	+ '032	— '189
Evening Observations, September 23.										
375	63°	B	b	^r 138°6725	3567''82	3567''78	+ 0''04	+ '973	+ '067	+ '193
376	60	A	c	142°0113	3653'57	3654'14	— 0'57	— '336	+ '939	— '058
377	57	B	μ	217°8219	5603'82	5603'69	+ 0'13	— '974	+ '041	— '178
378	51	B	μ	218°2105	5613'82	5613'82	0'00	— '974	+ '041	— '165
379	46	B	b	137°3923	3534'88	3535'24	— 0'36	+ '973	+ '068	+ '156
Evening Observations, September 24.										
380	60°	A	c	^r 153°2112	3941''70	3941''84	— 0''14	— '453	+ '885	— '080
381	56	A	h	115°8913	2981'75	2983'72	— 1'97	+ '970	+ '099	+ '178
382	52	A	a	152°9594	3935'27	3936'52	— 1'25	+ '873	— '448	+ '152
383	39	A	β	210°2576	5409'10	5408'77	+ 0'33	— '812	— '555	— '106
384	37	B	β	210°3963	5412'67	5412'00	+ 0'67	— '813	— '555	— '100
385	32	B	h	114°2209	2938'77	2939'99	— 1'22	+ '970	+ '102	+ '113
Morning Observations, September 24.										
388	36°	B	β	^r 214°5438	5519''38	5518''36	+ 1''02	— '821	— '541	+ '110
389	39	B	b	109°0635	2806'08	2807'61	— 1'53	+ '969	+ '112	— '132
390	47	A	c	157°6712	4056'56	4056'75	— 0'19	— '488	+ '865	+ '076
391	54	A	a	145°7385	3749'51	3749'50	+ 0'01	+ '864	— '465	— '153
392	55	B	β	215°6029	5546'63	5546'47	+ 0'16	— '823	— '538	+ '148

* The groups 369 and 370 were observed without the use of the reversing prism.

TABLE II.—*continued.*

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Distance in Terms of the Screw.	Observed Distance Corrected for Phase.	Tabular Distance of Centre of Mars from Star.	O—C.	Coefficients of		
								Δa f'	Δb f''	Δc f'''
Morning Observations, September 25.										
397	41°	A	c	171'2779	.4406"51	4406"41	+0"10	—·565	+·815	+·077
398	61	A	a	125'7885	3236"25	3235"90	+0"35	+·838	—·512	—·168
399	64	A	β	231'9950	5968"33	5967"54	+0"79	—·854	—·485	+·162
400	68	A	b	87'4161	2249"20	2249"51	—0"31	+·959	+·181	—·190
Evening Observations, September 27.										
401	52°	A	b	61'1494	1573"49	1573"27	+0"22	+·904	+·374	+·153
402	50	A	β	255'3711	6569"72	6569"25	+0"47	—·890	—·413	—·144
403	46	A	c	196'6325	5058"79	5058"66	+0"13	—·646	+·748	—·101
404	42	A	a	96'4914	2482"50	2482"67	—0"17	+·787	—·593	+·117
405	39	B	a	96'3594	2479"11	2478"67	+0"44	+·786	—·593	+·111
Evening Observations, October 3.*										
426	54°	A	a	15'6048	401"45	401"18	+0"27	+·285	—·956	+·044
427	44	A	b	67'7476	1742"98	1742"92	+0"06	—·248	+·967	—·038
428	33	A	b	68'0382	1750"46	1750"71	—0"25	—·252	+·966	—·036
429	28	A	a	14'7706	379"98	379"65	+0"33	+·247	—·968	+·033
Morning Observations, October 3.*										
430	25°	A	a	13'0648	336"06	336"48	—0"42	+·155	—·987	—·002
431	31	A	b	70'1930	1805"89	1806"22	—0"33	—·273	+·960	+·019
432	36	A	b	70'3777	1810"64	1811"17	—0"53	—·275	+·960	+·026
433	56	A	a	12'0819	310"75	310"92	—0"17	+·090	—·995	—·019
434	60	A	a	11'9384	307"06	307"05	+0"01	+·081	—·996	—·021
435	65	A	b	71'4141	1837"30	1837"93	—0"63	—·281	+·957	+·061

* The evening and morning observations of October 3 were intended as a trial of the value of position angles as a means of determining parallax. The observations of distance are valuable for determining the tabular error in Declination.

The equations resulting from the Position Angles will be found on the next page.

In the preceding Table the reader will observe that the various 'Groups' are arranged in 'Bunches' of two, three, or more groups; and I would here define a 'Bunch' of equations, as *two, three, or more equations, resulting from a corresponding number of groups of observations, which have all been made so closely together, that, in their discussion, the scale value may be considered uniform throughout the 'Bunch.'*

The following Table is similarly formed from the measures of Position Angle.

TABLE III.
Comparison of Observed and Tabular Position Angles.

Group.	Mean Zenith Distance of Group.	Mode of Observation.	Star of Comparison.	Observed Position Angle.	Computed Pos. Angle of Centre of <i>Mars</i> from Star.	O-C.	Coefficients of			
							$\Delta\alpha$	$\Delta\delta$	n	
<i>Evening Observations, October 3.</i>										
426	54°	A	<i>a</i>	163° 2'16 + k_1 *	163° 5'99	-3'83 + k_1	-8.01	-2.50	-1.32	
427	44	A	<i>b</i>	345 16.79 + k_1	345 13.28	+3.51 + k_1	+1.86	+0.50	+0.25	
428	33	A	<i>b</i>	345 6.90 + k_2	345 1.98	+4.92 + k_2	+1.85	+0.50	+0.20	
429	28	A	<i>a</i>	165 17.45 + k_2	165 20.78	-3.33 + k_2	-8.55	-2.29	-0.75	
<i>Morning Observations, October 3.</i>										
430	25°	A	<i>a</i>	170° 58'47 + k_3	171° 3'62	-5'15 + k_3	-9.87	-1.59	+0.85	
431	31	A	<i>b</i>	343 48.99 + k_3	343 45.12	+3.87 + k_3	+1.78	+0.53	-0.19	
432	36	A	<i>b</i>	343 42.23 + k_4	343 39.43	+2.80 + k_4	+1.78	+0.53	-0.21	
433	56	A	<i>a</i>	174 36.33 + k_4	174 42.24	-5.91 + k_4	-10.75	-1.02	+1.80	
434	60	A	<i>a</i>	175 8.95 + k_5	175 14.90	-5.95 + k_5	-10.87	-0.94	+1.91	
435	65	A	<i>b</i>	343 18.12 + k_5	343 13.59	+4.53 + k_5	+1.75	+0.54	-0.32	

* k_1, k_2, k_3 , &c., represent the Index Error of the Position Circle. This Index Error is not supposed constant, it being assumed that it may vary from flexure, or from errors of adjustment in the Equatorial mounting.

CHAPTER XIV.

FORMATION OF THE EQUATIONS OF CONDITION.

The best mode of discussing a large mass of material like that contained in Chapter XIII., Table II., is probably, to some extent, a matter of opinion. It is very desirable, therefore, to consider the various methods which *may* be adopted, before we attempt to arrive at any definitive ideas on the subject.

In a series of papers which I communicated to "*The Observatory*" on "*the determination of the Solar Parallax*," I arrived, by a course of reasoning which, so far as I know, has not yet been disputed, at the following conclusions:—

- "No method can be admitted which is in the remotest degree liable to systematic error; and every method in which the errors are not self-correcting is liable to such systematic error. That is to say, the observations themselves must afford the material for the determination and elimination of all errors to which it can be shown the method is liable."* And
- "The method must rest on a simple system of rigid accuracy both as to observation and reduction, so that every theorist or computer must, from the same series of observations, arrive at precisely the same result."*

These tests and conditions I propose to apply to the following discussion.

The first step is to form a list of all conceivable errors that may affect the comparison between an observed and tabular distance beyond the quantities $\Delta\alpha$, $\Delta\delta$, and n , which we desire to determine. So far as I can judge, the whole of such errors are comprised in the following list.

- | | | |
|---|---|---|
| C | { | <ol style="list-style-type: none"> 1. Errors of the adopted tabular place of the comparison star. 2. Errors, due to error of the tabular motion in the interval from the common epoch for which the values of $\Delta\alpha$ and $\Delta\delta$ are to be deduced.* |
|---|---|---|

* It would be hypercritical to include the possible errors of the tabular value of Δ ; for our purposes these *must* be insensible.

- O {
3. Error due to personality in making the observations, such, for example, as a tendency to place the star (in observing by Method A) not in the true centre of the planet, but persistently in some other position, either too high or low, or too much to right or left; or a tendency (in observing by Method B) to bring a star further on the faint limb than on the bright one.*
 4. Error of the value of the screw, due either to an error of the mean value adopted, or to an abnormal value for the time being (caused by difference of temperature in the different parts of the instrument, or by an abnormal state of the observer's eye).
 5. Error due to the Refraction Tables, by which the refractions, to correct the observed distances, were computed; or to abnormal refraction at the time of observation.†
 6. Error of the determination of the corrections of the scale or screw.
 7. The unavoidable accidental error of bisection.‡

Of these errors, the two first affect the tabular quantities, the remainder affect the observed quantities.

Discussing them in order:

1. The errors of the adopted place of the comparison star are easily eliminated if the errors under class 2 may be neglected. It then only becomes necessary to employ the same stars in the evening and morning observations, when the errors of the star-places disappear in computing the parallax. This, so far, is sufficiently simple, but the exclusive employment of symmetrical observations involves the rejection of a good many observations.§

* The limb which is full is always sensibly brighter than that affected by phase, and different areas of the surface of the planet are of very different brilliancy.

† The latter, of course, ranks only as an accidental error.

‡ It is quite unnecessary to include, as a possible source of systematic error, variations in the rate of the chronometer by night and by day. At the period of greatest tabular motion the change of tabular place only amounts to $0''.01$ of arc in a second of time, and the possibility of such an error occurring is entirely excluded by the remarks at end of Chapter III.

§ It is, of course, easy enough to plan observations of the required symmetry. Such plans of observation were prepared for every morning and evening, but any one who attempts to carry out such a series of observations will very soon find how different it is to propose and to execute.

Here then the question is opened whether it is better to eliminate the errors of the star places by the employment of only strictly symmetrical observations, or to undertake a general discussion in which the errors of the star places are treated as accidental errors of the equations, and *every* observation is employed in the parallax determination. I have consulted many of the best authorities, and I have received every shade of opinion. I quote three of these opinions:—

- (a) "I don't believe in any results where observations are rejected: you must include every observation made, or otherwise I cannot put any confidence in the result."
- (b) "It is impossible to say which would be the better method until the material is before you—but either method is sound."
- (c) "I don't believe in any determination of parallax in which the star places are considered to be absolutely known—unless the same stars are used morning and evening I don't consider the result worth having."

For my own part, I think there is much to be said on both the extreme sides of the question, and I agree *in toto* with the opinion *b*; therefore I propose to give the results by both methods; but there are considerations, to which I shall afterwards refer, and which had not presented themselves to my mind at the time when I received the opinions above quoted, which have induced me to select, as the definitive method, that in which only symmetrical evening and morning observations are used. For the latter method only, then, shall I give the detailed steps of reduction, but I shall quote sufficiently from the reductions by the first method to enable the reader to judge of the comparative merits of the two systems.

2. The errors of tabular motion can be eliminated by considering the star places to be known, and deducing, from the values of $\Delta\alpha$ and $\Delta\delta$ for different epochs, the actual change of tabular error. The observations extend over so long a period, and the parallax factors have opposite signs in such short intervals, that no conceivable error in the star-places (whence the values of $\Delta\alpha$ and $\Delta\delta$ are derived) could sensibly affect the final result, except, perhaps, by slightly increasing its probable error. To deal with the matter, it

is only necessary to introduce on the right hand side of the equation two terms expressing the effect on the distance of a daily increase of $\Delta\alpha=\kappa$ and of a daily increase of $\Delta\delta=\kappa'$.*

Then having found the value of n in terms of κ and κ' , from each combination of observations, and finally the values of $\Delta\alpha$ and $\Delta\delta$ for each epoch τ_0 , the values of the daily change of $\Delta\alpha$ and $\Delta\delta$ for each epoch (i.e. of κ and κ') become known, and thence the absolute values of n .

3. The errors which have their origin in personality of the observer are nearly all eliminated by proper use of the little prism of reversion referred to in Chapters II. and VI. At least, so far as I can see, they are so eliminated in Method A.† In Method B, however, I consider this more doubtful. It is quite possible, and indeed *à priori* probable, that the personality of bisection of the true visible limb of the planet may be different for opposite limbs, if one limb is brighter than the other. It is probable also that this effect will be greater in the case of very faint stars than in bright stars, because the former are lost sight of upon the disk of the planet. With a very faint star there would probably be a tendency to place the star farther from the centre of the planet when the limb is very bright than when the limb is faint. The effect of such personality would be to make the absolute place of the planet different when faint stars are observed from that when bright stars are employed. Thus, if these suppositions are correct, the determination of $\Delta\alpha$ and $\Delta\delta$ would be affected by systematic error (particularly when faint stars are used), *but there would be no systematic error in the resulting parallax if the same stars are employed in the evening and morning observations.* This appears to be a strong argument for the employment of rigidly symmetrical morning and evening observations in the definitive discussion of the parallax.

4. With regard to the scale value. If the distance of the planet is

* The coefficient of κ will be $f'(\tau-\tau_0)$ and that of κ' , $f''(\tau-\tau_0)$ where f' and f'' are the factors in R.A. and Dec. respectively, τ_0 the mean epoch for which it is desired to find the value of $\Delta\alpha$, and τ the epoch of observation.

† I do not mean to assert that such measures are made, or can be made, without any absolute systematic error. I can readily believe that, when the planet has considerable phase, the estimation of the centre, even with the use of the reversing prism and with the precautions described in the *Monthly Notices*, vol. xxxix. p. 60, may yet be affected with an absolute systematic error, but it will be an error which will be the same for all stars, and one that can afterwards be developed in terms of the time from δ .

measured from nearly equidistant and opposite stars, the parallax resulting from the combined evening and morning observations is free from errors of the scale value, because these errors have equal and opposite effect when the calculations are made separately from the two stars. This general proposition is equally true, if, *in the mean*, the distances have been observed as often, and in as great extent, from stars preceding as from stars following the planet. From this it follows that if equations are formed with an erroneous scale value (or including terms which express, in the resulting value of n , the effect of an erroneous scale value), the result of their solution becomes the best test whether the condition has been fulfilled that the series must be one "*in which the errors are self-correcting*"—that is, "one in which the observations themselves afford the material for the elimination of these errors."

If then we assume, *firstly*, that the heliometer scale-value is not a constant quantity, and if we introduce into each combination of equations terms depending upon an unknown scale-value, and determine that scale-value from each combination (or, what is the same thing, eliminate these terms before solving for the remainder of the unknown quantities), we shall thus obtain the best possible solution of the problem.

And if, *secondly*, we assume that the heliometer scale-value is really constant, and that the apparent changes of scale-value are in reality only errors of measurement combined with errors in the adopted places of the stars of comparison, and therefore solve the equations, omitting the terms depending on change of scale-value, we obtain a *second* result.

The precision of the agreement of the *first* and *second* results will afford a gauge of the perfection of the arrangement of the combinations for eliminating systematic error due to scale-value; whilst the comparison of the probable errors of the two results will indicate which of the two methods of treatment was the better. I have accordingly carried out the solution by both methods in the following Chapter.

5. With regard to the Refraction. The tables and methods by which the refractions have been computed give rigidly the results of BESSEL'S Refraction Tables. For reasons already explained in Chapter VII., I have not pushed the measures of *Mars* in any case to extreme zenith distances, and the reader, by running his eye over the mean Z.D.'s of each group, as given

in Chapter XIII., Table II., Column 2, will see that in no case does the zenith distance exceed 72° . The greatest refraction-correction is less than $9''$, and this is an extreme case; in general the refraction-correction is less than half of this.

It is tolerably certain that for zenith distances from 0° to 72° the *law* of refraction is known, and that the constant of the law, in BESSEL's Tables, is not $\frac{1}{100}$ th part in error. Thus, in the most extreme case, the largest conceivable error of the tables would only affect the most extreme distance measured by $0''.09$. Of course if this error were systematic it would be entirely fatal, but here again the symmetrical arrangement of the stars of comparison comes to our aid, since errors of the refraction will produce opposite effects on the resulting parallax, according as the star of comparison precedes or follows the planet.

In order, however, to obtain a mathematical determination of the extent of freedom of the final result from errors of refraction, I have introduced terms into the equations which express the effect of increasing the tabular refraction $\frac{1}{100}$ th part; and, to make the test still more complete, I have supposed that the error of BESSEL's Refraction Tables may be different E. and W. of the meridian, for observations at Ascension Island.

There is, however, one possible source of error—an effect of refraction—which cannot be eliminated in so easy and simple a manner; it was first suggested to me by Sir GEO. AIRY, but, unfortunately, not until after the Expedition had taken place.

Since the light of a star is not monochromatic, and the dispersive power of air is very sensible, the image of a star, viewed at a considerable zenith distance, is not a disk but a spectrum. AIRY has shown (*Monthly Notices*, vol. xxix. p. 337) that the length of this spectrum, reckoned from B to G (which lines may be fairly taken to represent the visible length of the spectrum), is about $\frac{1}{8}$ of the atmospheric refraction.

Tabulating the length of this spectrum, we have—

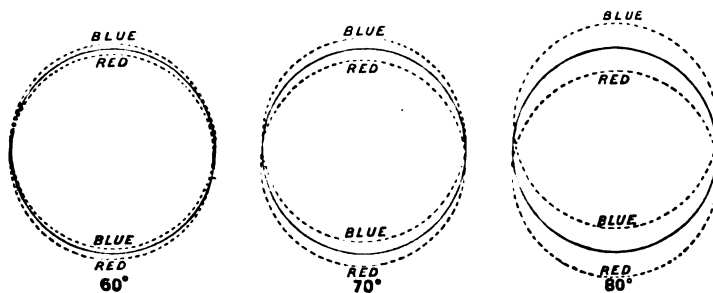
Z.D.	30°	$0''.56$
	40	$0''.81$
	50	$1''.15$
	60	$1''.68$
	70	$2''.65$
	80	$5''.59$

Now let us consider the effect of this on astronomical observations. If all observers could always select the same part of this spectrum (say, for example, the immediate neighbourhood of the line D) for bisection, then dispersion would have no influence on atmospheric refractions, and the law of refraction would be determined for rays of the refrangibility of D. As a matter of fact, observers do select very nearly the same part of the spectrum (somewhere between D and E) for observation, and hence the comparatively satisfactory agreement of independent refraction tables, at least so far as Z.D. 70° .

But in the case of a planet the matter is much more complicated. A diagram will enable the reader to realise the conditions.

The following diagram gives the limits of the dispersion of the limb of *Mars*, drawn to scale for semi-diameter $12''$, and for each of the zenith distances 60° , 70° , and 80° . The solid line represents the true limb of the planet, the dotted lines show its visible limits as affected by the chromatic dispersion of the atmosphere.

Fig. 4.

Limits of Atmospheric Chromatic Dispersion of Limb of *Mars*.

If the observer could make sure of bisecting the middle part of the coloured fringe, no systematic error in the observation of the planet would occur. But the reader, by reference to the diagram, will perceive how very difficult this becomes in case of a planet, because the blue of the lower limb will be projected on the body of the planet, whilst the blue of the upper limb is projected upon the background of the sky. It is therefore an open question whether observations are not liable to some systematic error on this

account. Had my attention been directed to this subject before the Expedition, I should have made special observations at very great and at very small zenith distances to test this point; but, as it is, I shall employ the only test at my disposal, and divide the observations into combinations of greater and lesser zenith distance, and deduce the parallax independently from both.

6 and 7. So much care has been bestowed on the determination of the errors of the scale and screw, and the errors of the scale have been so completely discussed in Chapter IX. in connection with the observation of distances determined by meridian observation (with the result that no systematic error could be found in the division-error determination), that I think any residual errors of the scale and screw may be considered purely accidental, and such as to exercise no *systematic* influence on the final result.

Summarising these remarks, we find that the final equations must be found as follows :—

The true distance = C	{ The tabular distance computed from the tabular R.A., Dec., and parallax of the planet, and the definitive apparent place of the star.
+ f' Δα	{ The corrections of the tabular distance due to tabular errors in R.A. and Dec. of planet.
+ f'' Δδ	
+ f''' n	{ The correction of the tabular distance due to tabular error of the parallax.
+ f'(τ - τ₀) κ	{ The corrections for change of Δα and Δδ between epoch of observation and epoch for which it is desired to find Δα and Δδ.
+ f''(τ - τ₀) κ'	

And also

The true distance = O	{ The observed distance calculated by the methods and with mean screw value already given.
+ vz	{ Correction for error of screw value, where z is the required cor- rection in 10,000'', and $v = \frac{\text{observed distance in seconds of arc}}{10,000}$.
+ rm	{ The correction for systematic error of BESSEL's Refraction Tables where $r = \frac{\text{the refraction in seconds of arc}}{100}$.
± ε	{ The accidental errors of measure and out-standing accidental errors of scale and screw.

Equating these two expressions of the true distance, we have—

$$C + f' \Delta \alpha + f'' \Delta \delta + f''' n + f'(\tau - \tau_0) \kappa + f''(\tau - \tau_0) \kappa' = O + vz + rm \pm \epsilon$$

and transposing, we get the following equation, which is the type upon which all other equations must be founded,

$$(1) \quad f'\Delta\alpha + f''\Delta\delta + f'''n - vz = (O-C) + rm - f'(\tau - \tau_0) \kappa - f''(\tau - \tau_0) \kappa'.$$

For the actual formation of these equations, we obtain from Chapter XIII., Table II., Columns 9, 10, and 11, the factors f' , f'' , and f''' respectively; and column 8 of the same Table gives the value of $(O-C)$.

Since, in forming equations on the type of equation (1), it is assumed that κ and κ' vary proportionally with the time, it is necessary that only such observations should be combined as have been made at epochs sufficiently near together to render this a safe assumption. Having regard to the perfection of the Tables of *Mars*, it may be considered practically safe to include, in the same combination, equations resulting from observations made within twenty-four hours of the mean epoch τ_0 .

But with regard to the term z , as we shall afterwards see, it is desirable to limit the period of its supposed constancy as much as possible, and therefore we must consider z different for each "bunch" of equations (Chapter XIII., Table II.).

Also it is desirable to discuss what the effect would be on the final result if the constant of refraction were different E. and W. of the meridian, and therefore in equations from evening observations we write the refraction term m , and in the morning observations m' .

Thus we shall have

$\Delta\alpha$, $\Delta\delta$, κ , and κ' common to all the equations of the same Combination.

z , z' , z'' , and z''' common only to equations of the same 'Bunch' in which they occur.

but n , m , and m' common to all the Combinations.

Following these general principles, the following Table I. has been prepared, containing all the complete combinations that can be framed subject to conditions of strict symmetry in the morning and evening observations. This symmetry must, of course, include an identity of the mode in which the corresponding observations were made, as well as symmetry in the arrangement of the equations.

In some cases, as, for example, in combination viii., the equation 151

has been used twice, once in bunch xxvi. and once in bunch xxviii. Again the equation 68 may equally well belong to bunch ix. or xi., because it may equally well be considered to be founded on the last group of observations of bunch xi. or on the first group of bunch ix.; and the equation 92 properly belongs to bunch xiv. and not to bunch xii. at all, the value of z'' must therefore be determined for group xiv., and substituted in equation 92 before the solution of group xii.

Every case where one observation has been used twice is marked with an asterisk, and in some cases it has been necessary to take the mean of two equations for the sake of symmetry. It appeared to me better to do this than to reject one of the observations. There are in all 216 equations, and, amongst these, 6 result from the mean of 2 groups; and 5 of the original equations have been used twice.

Such trifling departure from a rigid system of weights seemed to me preferable to rejecting a bunch of equations or a whole combination on the one hand, or to sacrificing the symmetry of the morning and evening equations on the other; and it is certain that the departure in question from a rigid system can have no sensible effect on the resulting weight or value of the parallax.

In some cases, chiefly between September 5 and 11, when the planet was very favourably situated between two stars e and g , of nearly the same declination, all the observer's attention, during the most favourable time, was devoted to the securing of measures of the planet's distance from these stars. Thus in the combinations from xiv. to xxi. inclusive, the equation in n contains $\Delta\delta$ on the right hand side of the equation, though with a very small factor, and the absolute value of $\Delta\delta$ is obtained with all needful accuracy from the general expression (or curve) for $\Delta\delta$. In two other cases, when the planet was nearly in a line between two stars, the values of $\Delta\alpha$ and $\Delta\delta$ with very small factors also appear on the right hand side of the equations in n .

TABLE I.

COMBINATION I.

July 31, Evening and Morning.

Method of Observation A.

Mean Interval from $\delta - 35^{\circ} 88$.

GREATER ZENITH DISTANCES.

Evening Observations.

i. $\zeta_0 = 59^{\circ}$	q	42	$+ \cdot 18 \Delta a + \cdot 98 \Delta \delta + \cdot 05 n - \cdot 70 z = + \cdot 08 i + \cdot 02 m + \cdot 02 k + \cdot 15 k'$
	n	43	$+ \cdot 98 + \cdot 14 + \cdot 17 - \cdot 44 = + \cdot 017 + \cdot 05 + \cdot 13 + \cdot 01$
	t	44	$- \cdot 90 - \cdot 40 - \cdot 15 - \cdot 42 = + \cdot 006 + \cdot 03 - \cdot 10 - \cdot 04$

Morning Observations.

ii. $\zeta_0 = 34^{\circ}$	t	51	$- \cdot 90 \Delta a - \cdot 41 \Delta \delta + \cdot 09 n - \cdot 42 z' = + \cdot 08 i + \cdot 01 m' + \cdot 11 k + \cdot 04 k'$
	n	52	$+ \cdot 98 + \cdot 13 - \cdot 11 - \cdot 44 = - \cdot 037 + \cdot 02 - \cdot 13 - \cdot 02$
	q	53	$+ \cdot 19 + \cdot 98 - \cdot 02 - \cdot 70 = + \cdot 110 + \cdot 02 - \cdot 03 - \cdot 14$

LESSER ZENITH DISTANCES.

Evening Observations.

iii. $\zeta_0 = 43^{\circ}$	t	45	$- \cdot 90 \Delta a - \cdot 40 \Delta \delta - \cdot 14 n - \cdot 42 z'' = + \cdot 054 + \cdot 02 m - \cdot 09 k - \cdot 04 k'$
	n	46	$+ \cdot 98 + \cdot 14 + \cdot 13 - \cdot 44 = + \cdot 017 + \cdot 02 + \cdot 08 + \cdot 01$
	q	47	$+ \cdot 19 + \cdot 98 + \cdot 03 - \cdot 70 = + \cdot 042 + \cdot 02 + \cdot 01 + \cdot 07$

Morning Observations.

iv. $\zeta_0 = 21^{\circ}$	q	48	$+ \cdot 19 \Delta a + \cdot 98 \Delta \delta - \cdot 02 n - \cdot 69 z''' = + \cdot 053 + \cdot 02 m' - \cdot 02 k - \cdot 08 k'$
	n	49	$+ \cdot 98 + \cdot 13 - \cdot 07 - \cdot 44 = + \cdot 090 + \cdot 02 - \cdot 09 - \cdot 01$
	t	50	$- \cdot 90 - \cdot 41 + \cdot 08 - \cdot 42 = + \cdot 075 + \cdot 01 + \cdot 10 + \cdot 05$

COMBINATION II.

August 5, Evening and Morning.

Method of Observation A.

Mean Interval from $\delta - 30^{\circ} 92$.

GREATER ZENITH DISTANCE.

Evening Observations.

v. $\zeta_0 = 66^{\circ}$	q	55	$+ \cdot 32 \Delta a + \cdot 95 \Delta \delta + \cdot 08 n - \cdot 67 z'' = + \cdot 026 + \cdot 04 m + \cdot 04 k + \cdot 12 k'$
	t	56	$- \cdot 80 - \cdot 59 - \cdot 16 - \cdot 37 = + \cdot 050 + \cdot 05 - \cdot 10 - \cdot 07$
	n	57	$+ \cdot 99 + \cdot 02 + \cdot 18 - \cdot 52 = - \cdot 002 + \cdot 06 + \cdot 11 \cdot 00$

Morning Observations (only group).

vi. $\zeta_0 = 30^{\circ}$	q	61	$+ \cdot 32 \Delta a + \cdot 95 \Delta \delta - \cdot 03 n - \cdot 67 z''' = - \cdot 005 + \cdot 02 m' - \cdot 04 k - \cdot 11 k'$
	t	62	$- \cdot 79 - \cdot 59 + \cdot 07 - \cdot 37 = - \cdot 003 + \cdot 01 + \cdot 10 + \cdot 08$
	n	63, 64	$+ \cdot 99 + \cdot 02 - \cdot 13 - \cdot 52 = + \cdot 002 + \cdot 02 - \cdot 17 \cdot 00$

TABLE I.—continued.

COMBINATION II.—continued.

LESSER ZENITH DISTANCE.

Evening Observations.

vii. $\zeta_0 = 55^\circ$	n	58	$+ '99\Delta\alpha + '02\Delta\delta + '18n - 52'' = -0.48 + '05m + '10\kappa + '00\kappa'$
	t	59	$- '80 - '59 - '14 - '37 = +0.55 + '03 - '07 - '05$
	q	60	$+ '32 + '95 + '06 - 67 = +0.59 + '03 + '04 + '08$

COMBINATION III.

August 12, Evening; August 10, Morning.

Method of Observation A.

Mean Interval from $\delta - 24^h 89$.

GREATER ZENITH DISTANCE.

Evening Observations, August 12.

viii. $\zeta_0 = 63^\circ$	q	71	$+ '27\Delta\alpha + '96\Delta\delta + '07n - '53z = -0.18 + '02m - '22\kappa - '80\kappa'$
	t	72	$- '72 - '68 - '15 - '51 = +0.12 + '05 + '60 + '57$
	n	73	$+ '95 - '25 + '18 - '46 = +0.13 + '04 - '80 + '21$

Morning Observations, August 10.

ix. $\zeta_0 = 52^\circ$	q	68*	$+ '31\Delta\alpha + '95\Delta\delta - '05n - '57z' = +0.07 + '02m' + '26\kappa + '80\kappa'$
	t	69	$- '73 - '68 + '12 - '46 = +0.57 + '02 - '62 - '58$
	n	70	$+ '97 - '16 - '17 - '49 = +0.18 + '04 + '81 - '13$

LESSER ZENITH DISTANCE.

Evening Observations, August 12.

x. $\zeta_0 = 53^\circ$	n	74	$+ '95\Delta\alpha - '25\Delta\delta + '17n - '46z'' = +0.11 + '04m - '81\kappa + '21\kappa'$
	t	75	$- '72 - '68 - '13 - '51 = +0.41 + '03 + '62 + '58$
	q	76	$+ '27 + '96 + '05 - 53 = -0.12 + '02 - '23 - '84$

Morning Observations, August 12.

xi. $\zeta_0 = 63^\circ$	n	66	$+ '97\Delta\alpha - '16\Delta\delta - '14n - '49z''' = +0.33 + '02m' + '83\kappa - '14\kappa'$
	t	67	$- '72 - '68 + '11 - '45 = +0.39 + '02 - '61 - '58$
	q	68*	$+ '31 + '95 - '05 - '57 = +0.07 + '02 + '26 + '80$

TABLE I.—*continued.*

COMBINATION IV.

August 14, Evening; August 13 and 14, Morning.

Method of Observation B.

Mean Interval from $\delta - 22^h 15$.

GREATER ZENITH DISTANCE.

Evening Observations, August 14.

xii. $\zeta_0 = 59^\circ$	<i>t</i> 88	$-.73\Delta\alpha - .67\Delta\delta - .16n - .58z$	$= -.035 + .04m + .07\kappa + .06\kappa'$
	<i>n</i> 89	$+.90 - .39 + .17 - .42$	$= -.03 + .03 - .09 + .04$
	<i>q</i> 92*	$+.17 + .99 + .04 - .47z''$	$= -.109 + .02 - .02 - .12$

Morning Observations, August 13.

xiii. $\zeta_0 = 46^\circ$	<i>q</i> 78	$+.21\Delta\alpha + .98\Delta\delta - .03n - .49z'$	$= -.031 + .02m' + .13\kappa + .63\kappa'$
	<i>n</i> 80, 81	$+.92 - .34 - .15 - .43$	$= -.068 + .03 + .55 - .20$
	<i>t</i> 82	$-.73 - .67 + .13 - .56$	$= +.019 + .03 - .41 - .38$

LESSER ZENITH DISTANCE.

Evening Observations, August 14.

xiv. $\zeta_0 = 54^\circ$	<i>n</i> 90	$+.90\Delta\alpha - .39\Delta\delta + .17n - .42z''$	$= -.014 + .02m - .10\kappa + .04\kappa'$
	<i>t</i> 91	$-.73 - .67 - .14 - .59$	$= +.029 + .02 + .08 + .07$
	<i>q</i> 92*	$+.17 + .98 + .04 - .47$	$= -.109 + .02 - .02 - .12$

Morning Observations, August 14.

xv. $\zeta_0 = 40^\circ$	<i>q</i> 93	$+.15\Delta\alpha + .99\Delta\delta - .02n - .46z$	$= -.049 + .01m' - .05\kappa - .36\kappa'$
	<i>t</i> 94	$-.73 - .67 + .11 - .60$	$= -.040 + .02 + .28 + .25$
	<i>n</i> 95	$+.89 - .42 - .15 - .41$	$= -.028 + .02 - .36 + .17$

COMBINATION V.

August 15, Evening and Morning.

Method of Observation A.

Mean Interval from $\delta - 20^h 93$.

GREATER ZENITH DISTANCE.

Evening Observations.

xvi. $\zeta_0 = 55^\circ$	<i>q</i> 97	$+.10\Delta\alpha + .99\Delta\delta + .03n - .44z$	$= -.071 + .01m + .01\kappa + .12\kappa'$
	<i>s</i> 98	$-.59 - .80 - .12 - .46$	$= +.064 + .02 - .06 - .09$
	<i>n</i> 99	$+.86 - .48 + .15 - .39$	$= +.009 + .02 + .09 - .05$

Morning Observations (only set).

xvii. $\zeta_0 = 38^\circ$	<i>q</i> 103	$+.07\Delta\alpha + 1.00\Delta\delta - .01n - .43z'$	$= -.115 + .01m' - .01\kappa - .13\kappa'$
	<i>s</i> 104	$-.60 - .79 + .08 - .47$	$= +.086 + .02 + .08 + .11$
	<i>n</i> 105, 106	$+.84 - .52 - .14 - .39$	$= -.006 + .02 - .13 + .08$

TABLE I.—continued.

COMBINATION V.—continued.									
LESSER ZENITH DISTANCE.									
Evening Observations.									
xviii. $\zeta_0 = 47^\circ$	<i>n</i>	100	.	+ '86Δα	— '49Δδ	+ '15 <i>n</i>	— '39 <i>z</i> "	= — 0'48 + '02 <i>m</i>	+ '08 <i>κ</i> — '04 <i>κ'</i>
	<i>s</i>	101		— '59	— '80	— '10	— '46	= + 0'79 + '02	— '05 — '07
	<i>q</i>	102		+ '09	+ '99	+ '02	— '44	= — 0'54 + '01	+ '01 + '08
COMBINATION VI.									
Method of Observation B.									
August 16, Evening and Morning.									
GREATER ZENITH DISTANCE.									
Evening Observations.									
xix. $\zeta_0 = 55^\circ$	<i>q</i>	108		+ '01Δα	+ '00Δδ	+ '01 <i>n</i>	— '41 <i>z</i>	= — 0'77 + '01 <i>m</i>	'00 <i>κ</i> + '13 <i>κ'</i>
	<i>s</i>	109		— '62	— '79	— '12	— '51	= — 0'19 + '03	— '07 — '09
	<i>n</i>	110		+ '80	— '58	+ '14	— '37	= + 0'46 + '02	+ '08 — '06
Morning Observations (only set).									
xx. $\zeta_0 = 56^\circ$	<i>q</i>	144		— '04Δα	+ '00Δδ	+ '01 <i>n</i>	— '40 <i>z</i> '	= — 0'04 + '01 <i>m'</i>	+ '01 <i>κ</i> — '19 <i>κ'</i>
	<i>s</i>	115		— '63	— '77	+ '11	— '53	= + 0'52 + '03	+ '13 + '15
	<i>n</i>	116		+ '77	— '62	— '16	— '36	= + 0'39 + '03	— '16 + '13
LESSER ZENITH DISTANCE.									
Evening Observations.									
xxi. $\zeta_0 = 45^\circ$	<i>n</i>	111		+ '80Δα	— '59Δδ	+ '13 <i>n</i>	— '37 <i>z</i> "	= + 0'04 + '02 <i>m</i>	+ '07 <i>κ</i> — '05 <i>κ'</i>
	<i>s</i>	112		— '62	— '78	— '10	— '51	= — 0'38 + '02	— '05 — '06
	<i>q</i>	113		'00	+ '00	'00	— '41	= — 1'27 + '01	'00 + '08
COMBINATION VII.									
Method of Observation B.									
August 19, Evening and Morning.									
GREATER ZENITH DISTANCE.									
Evening Observations.									
xxii. $\zeta_0 = 63^\circ$	<i>m</i>	123		+ '43Δα	+ '90Δδ	+ '10 <i>n</i>	— '54 <i>z</i>	= — 0'70 + '03 <i>m</i>	+ '04 <i>κ</i> + '09 <i>κ'</i>
	<i>n</i>	124		+ '45	— '89	+ '08	— '34	= + 0'81 + '02	+ '05 — '09
	<i>r</i>	125		— '91	+ '37	— '18	— '45	= — 1'26 + '05	— '08 + '03

TABLE I.—*continued.*

COMBINATION VII.—continued.

Morning Observations (only set).

xxiii. $\zeta_0 = 47^\circ$	<i>m</i>	130	$+ \cdot 41\Delta\alpha + \cdot 91\Delta\delta - \cdot 06n - \cdot 52z' = -\cdot 013 + \cdot 02m' - \cdot 08\kappa - \cdot 18\kappa'$
	<i>n</i>	131	$+ \cdot 39 - \cdot 92 - \cdot 07 - \cdot 34 = +\cdot 085 + \cdot 01 - \cdot 08 + \cdot 19$
	<i>r</i>	132	$- \cdot 93 + \cdot 33 + \cdot 17 - \cdot 46 = -\cdot 063 + \cdot 03 + \cdot 21 - \cdot 08$

LESSER ZENITH DISTANCE.

Evening Observations.

xxiv. $\zeta_0 = 54^\circ$	<i>r</i>	126	$- \cdot 91\Delta\alpha + \cdot 37\Delta\delta - \cdot 17n - \cdot 45z'' = -\cdot 143 + \cdot 04m - \cdot 07\kappa + \cdot 03\kappa'$
	<i>n</i>	127	$+ \cdot 44 - \cdot 89 + \cdot 08 - \cdot 34 = +\cdot 082 + \cdot 01 + \cdot 03 - \cdot 06$
	<i>m</i>	128	$+ \cdot 43 + \cdot 90 + \cdot 08 - \cdot 54 = -\cdot 028 + \cdot 02 + \cdot 03 + \cdot 05$

COMBINATION VIII.

August 20, Evening; August 21, Morning.

Method of Observation B.

Mean Interval from $\delta - 14^h 92$.

GREATER ZENITH DISTANCE.

Evening Observations.

xxv. $\zeta_0 = 59^\circ$	<i>n</i>	134	$+ \cdot 27\Delta\alpha - \cdot 96\Delta\delta + \cdot 04n - \cdot 34z = +\cdot 028 + \cdot 01m + \cdot 18\kappa - \cdot 62\kappa'$
	<i>r</i>	135	$- \cdot 95 + \cdot 27 - \cdot 18 - \cdot 49 = -\cdot 106 + \cdot 05 - \cdot 62 + \cdot 18$
	λ	136	$+ \cdot 93 + \cdot 32 + \cdot 18 - \cdot 66 = +\cdot 101 + \cdot 05 + \cdot 60 + \cdot 20$

Morning Observations.

xxvi. $\zeta_0 = 64^\circ$	<i>n</i>	151*	$+ \cdot 04\Delta\alpha - 1 \cdot 00\Delta\delta \cdot 00n - \cdot 37z'' = +\cdot 082 + \cdot 01m' - \cdot 02\kappa + \cdot 59\kappa'$
	λ	154	$+ \cdot 94 + \cdot 30 - \cdot 19 - \cdot 57z' = +\cdot 028 + \cdot 06 - \cdot 65 - \cdot 21$
	<i>r</i>	155	$- \cdot 97 + \cdot 16 + \cdot 21 - \cdot 57z' = -\cdot 106 + \cdot 09 + \cdot 68 - \cdot 11$

LESSER ZENITH DISTANCE.

Evening Observations.

xxvii. $\zeta_0 = 51^\circ$	λ	137	$+ \cdot 93\Delta\alpha + \cdot 32\Delta\delta + \cdot 18n - \cdot 66z'' = +\cdot 019 + \cdot 05m + \cdot 59\kappa + \cdot 20\kappa'$
	<i>r</i>	138	$- \cdot 95 + \cdot 27 - \cdot 17 - \cdot 50 = -\cdot 141 + \cdot 03 - \cdot 59 + \cdot 17$
	<i>n</i>	139	$+ \cdot 26 - \cdot 96 + \cdot 04 - \cdot 34 = +\cdot 092 + \cdot 01 + \cdot 16 - \cdot 59$

Morning Observations.

xxviii. $\zeta_0 = 29^\circ$	<i>n</i>	151*	$+ \cdot 04\Delta\alpha - 1 \cdot 00\Delta\delta \cdot 00n - \cdot 37z'' = +\cdot 082 + \cdot 01m - \cdot 02\kappa + \cdot 59\kappa'$
	<i>r</i>	152	$- \cdot 97 + \cdot 17 + \cdot 11 - \cdot 56 = -\cdot 084 + \cdot 02 + \cdot 58 - \cdot 10$
	λ	153	$+ \cdot 94 + \cdot 30 - \cdot 12 - \cdot 57 = +\cdot 072 + \cdot 02 - \cdot 57 - \cdot 18$

TABLE I.—continued.

COMBINATION IX.

August 24, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta - 11^h 91$.

GREATER ZENITH DISTANCE.

Evening Observations.

xxix. $\zeta_0 = 56^\circ$	n	171	$-.35\Delta\alpha$	$-.94\Delta\delta$	$-.09n$	$-.48z$	$= +0.27 + .03m$	$-.06\kappa$	$-.18\kappa'$
	q	172	$-.92$	$+.34$	$-.17$	$-.51$	$= -0.10 + .04$	$-.14$	$+.05$
	l	173	$+.98$	$.00$	$+.17$	$-.55$	$= +0.15 + .03$	$+.14$	$.00$

Morning Observations.

xxx. $\zeta_0 = 56^\circ$	q	182	$-.93\Delta\alpha$	$+.31\Delta\delta$	$+.18n$	$-.53z'$	$= -0.11 + .03m'$	$+.14\kappa$	$-.05\kappa'$
	l	183	$+.98$	$-.02$	$-.19$	$-.52$	$= +0.25 + .05$	$-.16$	$.00$
	n	184	$-.39$	$-.92$	$+.07$	$-.51$	$= +0.94 + .02$	$+.06$	$+.15$

LESSER ZENITH DISTANCE.

Evening Observations.

xxxi. $\zeta_0 = 42^\circ$	l	174	$+.98\Delta\alpha$	$.00\Delta\delta$	$+.16n$	$-.54z''$	$= -0.08 + .03m$	$+.13\kappa$	$.00\kappa'$
	q	175	$-.92$	$+.34$	$-.14$	$-.51$	$= -0.25 + .02$	$-.11$	$+.04$
	n	176	$-.36$	$-.93$	$-.05$	$-.49$	$= +0.52 + .01$	$-.04$	$-.10$

Morning Observations.

xxxii. $\zeta_0 = 39^\circ$	q	177, 181	$-.93\Delta\alpha$	$+.31\Delta\delta$	$+.13n$	$-.52z'''$	$= -0.55 + .02m'$	$+.09\kappa$	$-.03\kappa'$
	l	178, 180	$+.98$	$-.01$	$-.15$	$-.52$	$= +0.12 + .02$	$-.11$	$.00$
	n	179	$-.39$	$-.92$	$+.06$	$-.50$	$= +0.02 + .02$	$+.04$	$+.10$

COMBINATION X.

August 29, Evening and Morning.

Method of Observation A.

Mean Interval from $\delta - 6^h 95$.

Evening Observations (only set).

xxxiii. $\zeta_0 = 59^\circ$	m	204, 205	$-.19n$	$-.52z$	$= -0.25 + .92\Delta\alpha$	$-.34\Delta\delta$	$+.05m$	$-.14\kappa$	$+.05\kappa'$
	f	206	$+.19$	$-.70$	$= -0.70 - .97$	$+.14$	$+.05$	$+.13$	$-.02$

Morning Observations.

xxxiv. $\zeta_0 = 43^\circ$	f	213	$-.15n$	$-.67z'$	$= -0.39 - .97\Delta\alpha$	$+.16\Delta\delta$	$+.03m'$	$-.12\kappa$	$+.02\kappa'$
	m	214, 215	$+.15$	$-.55$	$= +0.26 + .93$	$-.31$	$+.03$	$+.13$	$-.04$

TABLE I.—*continued.*

COMBINATION XI.

August 29, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta - 6^h 96$.

Evening Observations.

xxxv. $\zeta_0 = 48^\circ$	<i>h</i>	207	$+ \cdot 83\Delta\alpha + \cdot 54\Delta\delta + \cdot 15n - \cdot 50z''$	$= -0\cdot 93 + 0\cdot 3m + \cdot 09\kappa + \cdot 06\kappa'$
	λ	208	$- \cdot 57 - \cdot 82 - \cdot 10 - \cdot 08$	$= +0\cdot 95 \cdot 00 - \cdot 06 - \cdot 08$
	<i>m</i>	209	$- \cdot 92 + \cdot 33 - \cdot 15 - \cdot 53$	$= -0\cdot 08 + 0\cdot 03 - \cdot 08 + \cdot 03$

Morning Observations.

xxxvi. $\zeta_0 = 28^\circ$	λ	210	$- \cdot 66\Delta\alpha - \cdot 74\Delta\delta + \cdot 07n - \cdot 10z'''$	$= +0\cdot 64 \cdot 00m' + \cdot 07\kappa + \cdot 07\kappa'$
	<i>h</i>	211	$+ \cdot 82 + \cdot 55 - \cdot 10 - \cdot 48$	$= +0\cdot 02 + 0\cdot 02 - \cdot 09 - \cdot 06$
	<i>m</i>	212	$- \cdot 93 + \cdot 31 + \cdot 11 - \cdot 54$	$= -1\cdot 40 + 0\cdot 02 + \cdot 10 - \cdot 03$

COMBINATION XII.*

August 31, Evening and Morning.

Method of Observation A.

Mean Interval from $\delta - 4^h 95$.

GREATER ZENITH DISTANCE.

Evening Observations.

xxxvii. $\zeta_0 = 47^\circ$	<i>f</i>	212 bis	$+ \cdot 17n - \cdot 54z''$	$= -0\cdot 50 - 0\cdot 4\Delta\alpha + 30\Delta\delta + 0\cdot 3m + \cdot 12\kappa - \cdot 04\kappa'$
	<i>m</i>	213 bis	$- \cdot 16 - \cdot 68$	$= +0\cdot 24 + 0\cdot 96 - \cdot 17 + \cdot 04 - \cdot 11 + \cdot 02$

Morning Observations.

xxxviii. $\zeta_0 = 39^\circ$	<i>m</i>	218	$+ \cdot 13n - \cdot 70z''$	$= +0\cdot 10 + 0\cdot 6\Delta\alpha - \cdot 15\delta + 0\cdot 3m' + \cdot 11\kappa - \cdot 02\kappa'$
	<i>f</i>	219	$- \cdot 15 - \cdot 51$	$= +0\cdot 88 - 0\cdot 93 + \cdot 33 + 0\cdot 02 - \cdot 12 + 0\cdot 04$

LESSER ZENITH DISTANCE.

Evening Observations.

xxxix. $\zeta_0 = 39^\circ$	<i>m</i>	214 bis	$- \cdot 15n - \cdot 68z''$	$= +0\cdot 58 + 0\cdot 6\Delta\alpha - \cdot 17\Delta\delta + 0\cdot 3m - \cdot 10\kappa + \cdot 02\kappa'$
	<i>f</i>	215 bis	$+ \cdot 14 - \cdot 53$	$= -0\cdot 57 - 0\cdot 93 + \cdot 30 + 0\cdot 02 + 0\cdot 08 - 0\cdot 03$

Morning Observations.

xl. $\zeta_0 = 27^\circ$	<i>f</i>	216 bis	$- \cdot 09n - \cdot 52z'''$	$= +0\cdot 25 - 0\cdot 93\Delta\alpha + 32\Delta\delta + 0\cdot 2m' - 0\cdot 7\kappa + 0\cdot 3\kappa'$
	<i>m</i>	217 bis	$+ \cdot 11 - \cdot 70$	$= -0\cdot 05 + 0\cdot 97 - \cdot 16 + 0\cdot 03 + \cdot 10 - 0\cdot 02$

* All the evening observations of this combination are noted as extremely unsatisfactory from bad definition, whilst the morning observations were hurried to escape cloud. The combination might very properly be rejected, but upon the whole I have thought it may be well to allow $\frac{1}{4}$ weight to the resulting value of n .

TABLE I.—continued.

COMBINATION XIII.

September 4, 5, and 6, Evening; September 5 and 6, Morning.

Method of Observation B.

Mean Interval from $g + 0^h.09$.

GREATER ZENITH DISTANCE.

Evening, September 4.

xli.	$g \ 231$	$+ \cdot 87 \Delta \alpha + \cdot 45 \Delta \delta + \cdot 19 n - \cdot 09 z$	$= + 0^m.06 + 0^m.01 m + 1^m.06 \kappa + \cdot 55 \kappa'$
$\zeta_0 = 60^\circ$	$g \ 232$	$+ \cdot 87 \quad + \cdot 45 \quad + \cdot 18 \quad - \cdot 09$	$= + 1^m.26 + 0^m.01 \quad + 1^m.04 \quad + \cdot 54$

Evening, September 5.

xlii.	$g \ 241$	$- \cdot 64 \Delta \alpha + \cdot 75 \Delta \delta - \cdot 13 n - \cdot 02 z$	$= - 1^m.20 \quad 0^m.00 m - 0^m.13 \kappa + \cdot 16 \kappa'$
$\zeta_0 = 56^\circ$	$g \ 242$	$- \cdot 67 \quad + \cdot 73 \quad - \cdot 13 \quad - \cdot 02$	$= - 1^m.23 \quad 0^m.00 \quad - 0^m.13 \quad + \cdot 14$
	$g \ 243$	$- \cdot 71 \quad + \cdot 69 \quad - \cdot 13 \quad - \cdot 02$	$= - 1^m.02 \quad 0^m.00 \quad - 0^m.13 \quad + \cdot 13$

Evening, September 6.

xliii.	$g \ 261, 262^*$	$- \cdot 97 \Delta \alpha - \cdot 09 \Delta \delta - \cdot 19 n - \cdot 11 z$	$= - 0^m.43 + 0^m.01 m + 0^m.78 \kappa + \cdot 07 \kappa'$
$\zeta_0 = 55^\circ$			

Morning, September 5.

xliv.	$g \ 254$	$- \cdot 96 \Delta \alpha + \cdot 17 \Delta \delta + \cdot 16 n - \cdot 04 z$	$= - 1^m.12 \quad 0^m.00 m + \cdot 05 \kappa - \cdot 01 \kappa'$
	$g \ 255$	$- \cdot 96 \quad + \cdot 16 \quad + \cdot 16 \quad - \cdot 05$	$= - 0^m.38 \quad 0^m.00 \quad + \cdot 07 \quad - \cdot 01$
	$g \ 257$	$- \cdot 97 \quad + \cdot 15 \quad + \cdot 17 \quad - \cdot 05$	$= - 0^m.14 \quad 0^m.00 \quad + \cdot 11 \quad - \cdot 02$
	$g \ 258$	$- \cdot 97 \quad + \cdot 14 \quad + \cdot 19 \quad - \cdot 05$	$= - 0^m.71 \quad 0^m.00 \quad + \cdot 12 \quad - \cdot 02$
	$g \ 259$	$- \cdot 97 \quad + \cdot 13 \quad + \cdot 21 \quad - \cdot 05$	$= - 0^m.57 + 0^m.01 \quad + \cdot 14 \quad - \cdot 02$
$\zeta_0 = 53^\circ$			

Morning, September 6.

xlv.	$g \ 274$	$- \cdot 97 \Delta \alpha - \cdot 12 \Delta \delta + \cdot 19 n - \cdot 14 z$	$= - 0^m.92 + 0^m.01 m + 1^m.07 \kappa + \cdot 13 \kappa'$
$\zeta_0 = 53^\circ$			

LESSER ZENITH DISTANCE.

Evening, September 4.

xlvi.	$g \ 233$	$+ \cdot 87 \Delta \alpha + \cdot 45 \Delta \delta + \cdot 17 n - \cdot 09 z''$	$= + 1^m.04 + 0^m.01 m + 1^m.04 \kappa + 0^m.54 \kappa'$
$\zeta_0 = 54^\circ$	$g \ 234$	$+ \cdot 87 \quad + \cdot 46 \quad + \cdot 17 \quad - \cdot 09$	$= - 0^m.07 + 0^m.01 \quad + 1^m.04 \quad + 0^m.55$

Evening, September 5.

xlvii.	$g \ 244$	$- \cdot 73 \Delta \alpha + \cdot 67 \Delta \delta - \cdot 13 n - \cdot 02 z''$	$= - 1^m.02 \quad 0^m.00 m - 1^m.13 \kappa + \cdot 12 \kappa'$
$\zeta_0 = 47^\circ$	$g \ 245$	$- \cdot 75 \quad + \cdot 64 \quad - \cdot 13 \quad - \cdot 02$	$= - 1^m.25 \quad 0^m.00 \quad - 1^m.13 \quad + \cdot 12$
	$g \ 246$	$- \cdot 77 \quad + \cdot 62 \quad - \cdot 13 \quad - \cdot 02$	$= - 1^m.00 \quad 0^m.00 \quad - 1^m.13 \quad + \cdot 11$

TABLE I.—*continued.*

COMBINATION XIII.—continued.

Evening, September 6.

xlvi.	<i>g</i> 266	$-.97\Delta\alpha - .10\Delta\delta - .16n - .11z'' = -.017 + .01m + .81\kappa + .08\kappa'$
$\zeta_0 = 41^\circ$	<i>g</i> 267	$-.97 - .10 - .16 - .11 = -.010 + .01 + .81 + .08$

Morning, September 5.

.	<i>g</i> 247	$-.94\Delta\alpha + .28\Delta\delta + .04n - .04z''' = -.067 + .00m' - .01\kappa + .00\kappa'$
	<i>g</i> 248	$-.94 + .27 + .05 - .04 = -.074 + .00 - .01 + .00$
xlix.	<i>g</i> 249	$-.95 + .25 + .06 - .04 = -.052 + .00 + .00 + .00$
$\zeta_0 = 18^\circ$	<i>g</i> 250	$-.95 + .24 + .07 - .04 = -.132 + .00 + .01 + .00$
	<i>g</i> 251	$-.95 + .23 + .08 - .04 = -.131 + .00 + .02 + .00$
	<i>g</i> 252	$-.95 + .22 + .09 - .04 = -.096 + .00 + .03 - .01$

Morning, September 6.

l.	<i>g</i> 270*	$-.97\Delta\alpha - .12\Delta\delta + .09n - .13z''' = -.086 + .00m' + .99\kappa + .12\kappa'$
$\zeta_0 = 24^\circ$		

COMBINATION XIV.

September 6, Evening and Morning.

Method of Observation B.

Mean Interval from $\gamma + 1^h 01.$

Evening Observations.

li.	<i>g</i> * 261, 262	$-.97\Delta\alpha - .19n - .11z = -.043 + .09\Delta\delta + .01m - .12\kappa - .01\kappa'$
$\zeta_0 = 53^\circ$	<i>e</i> 263	$+ .97 + .18 - .59 = + .081 - .13 + .04 + .11 + .01$

Morning Observations.

lii.	<i>g</i> * 270	$-.97\Delta\alpha + .09n - .13z = -.086 + .12\Delta\delta + .00m' + .10\kappa + .01\kappa'$
$\zeta_0 = 30^\circ$	<i>e</i> 271	$+ .97 - .13 - .57 = + .093 - .12 + .02 - .13 - .02$

COMBINATION XV.

September 6, Evening and Morning.

Method of Observation B.

Mean Interval from $\gamma + 1^h 04.$

Evening Observations.

liii.	<i>e</i> 264	$+ .97\Delta\alpha + .17n - .59z = + .013 - .13\Delta\delta + .03m + .13\kappa - .02\kappa'$
$\zeta_0 = 46^\circ$	<i>h</i> 265	$-.97 - .16 - .32 = -.043 - .13 + .02 - .11 - .02$

Morning Observations.

liv.	<i>e</i> 272	$+ .97\Delta\alpha - .14n - .57z' = + .113 - .12\Delta\delta + .03m' - .10\kappa - .01\kappa'$
$\zeta_0 = 42^\circ$	<i>h</i> 273	$-.97 + .17 - .35 = -.086 - .11 + .02 + .13 - .02$

TABLE I.—continued.

COMBINATION XVI.

September 7, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta + 2^h 01$.

Evening Observations.

lv.	e 278	$+ \cdot 97 \Delta a + \cdot 17 n - \cdot 49 z = + \cdot 052 - \cdot 10 \Delta \delta + \cdot 03 m + \cdot 10 \kappa + \cdot 01 \kappa'$
$\zeta_0 = 43^\circ$	g 279, 280	$- \cdot 96 - \cdot 15 - \cdot 21 = - \cdot 034 + \cdot 17 + \cdot 01 - \cdot 09 - \cdot 01$

Morning Observations.

lvi.	e 286	$+ \cdot 97 \Delta a - \cdot 10 n - \cdot 47 z' = + \cdot 015 - \cdot 10 \Delta \delta + \cdot 02 m' - \cdot 10 \kappa - \cdot 01 \kappa'$
$\zeta_0 = 28^\circ$	g 287	$- \cdot 96 + \cdot 11 - \cdot 23 = - \cdot 038 + \cdot 17 + \cdot 01 + \cdot 11 + \cdot 01$

COMBINATION XVII.

September 8, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta + 2^h 97$.

Evening Observations.

lvii.	h 290	$- \cdot 98 \Delta a - \cdot 21 n - \cdot 50 z = - \cdot 041 + \cdot 01 \Delta \delta + \cdot 08 m - \cdot 12 \kappa - \cdot 00 \kappa'$
$\zeta_0 = 60^\circ$	e 291	$+ \cdot 98 + \cdot 20 - \cdot 40 = - \cdot 058 - \cdot 07 + \cdot 03 + \cdot 09 - \cdot 00$

Morning Observations (only set).

lviii.	e 301*	$+ \cdot 98 \Delta a - \cdot 13 n - \cdot 37 z' = + \cdot 004 - \cdot 06 \Delta \delta + \cdot 02 m' - \cdot 16 \kappa - \cdot 01 \kappa'$
$\zeta_0 = 37^\circ$	h 303	$- \cdot 98 + \cdot 15 - \cdot 53 = + \cdot 046 + \cdot 02 + \cdot 02 + \cdot 17 - \cdot 00$

Evening Observations.

lix.	h 295	$- \cdot 98 \Delta a - \cdot 16 n - \cdot 50 z'' = + \cdot 029 + \cdot 01 \Delta \delta + \cdot 03 - \cdot 06 \kappa - \cdot 00 \kappa'$
$\zeta_0 = 42^\circ$	e 296	$+ \cdot 98 + \cdot 15 - \cdot 40 = - \cdot 001 - \cdot 07 + \cdot 02 + \cdot 05 - \cdot 00$

COMBINATION XVIII.

September 8, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta + 3^h 02$.

Evening Observations.

lx.	g 292, 293	$- \cdot 96 \Delta a - \cdot 18 n - \cdot 31 z = + \cdot 037 + \cdot 19 \Delta \delta + \cdot 02 m - \cdot 12 \kappa - \cdot 02 \kappa'$
$\zeta_0 = 49^\circ$	e 294	$+ \cdot 98 + \cdot 17 - \cdot 40 = - \cdot 057 - \cdot 07 + \cdot 02 + \cdot 11 + \cdot 01$

Morning Observations.

lxi.	e 301*	$+ \cdot 98 \Delta a - \cdot 13 n - \cdot 37 z' = + \cdot 004 - \cdot 06 \Delta \delta + \cdot 01 m' - \cdot 11 \kappa - \cdot 01 \kappa'$
$\zeta_0 = 36^\circ$	g 302	$- \cdot 96 + \cdot 14 - \cdot 33 = + \cdot 022 + \cdot 19 + \cdot 01 + \cdot 12 + \cdot 02$

TABLE I.—*continued.*

COMBINATION XIX.

September 9, Evening and Morning.

Method of Observation B.

Mean Interval from $\delta + 4^{\text{h}} 07$.

Evening Observations.

lxii.	<i>e</i>	307	$+ \cdot 98 \Delta \alpha + \cdot 18 n - \cdot 30 z$	$= + \cdot 028 - \cdot 02 \Delta \delta + \cdot 02 m + \cdot 17 \kappa - \cdot 00 \kappa'$
$\zeta_0 = 48^\circ$	<i>g</i>	308	$- \cdot 96 - \cdot 17 - \cdot 41$	$= - \cdot 063 + \cdot 20 + \cdot 02 - \cdot 16 - \cdot 03$

Morning Observations.

	<i>g</i>	311	$- \cdot 96 \Delta \alpha + \cdot 13 n - \cdot 43 z'$	$= + \cdot 019 + \cdot 19 \Delta \delta + \cdot 02 m' + \cdot 06 \kappa + \cdot 06 \kappa'$
lxiii.	<i>e</i>	312	$+ \cdot 98 - \cdot 15 - \cdot 28$	$= + \cdot 055 \cdot 00 + \cdot 01 - \cdot 08 \cdot 00$
$\zeta_0 = 45^\circ$	<i>e</i>	313	$+ \cdot 98 - \cdot 17 - \cdot 27$	$= + \cdot 053 \cdot 00 + \cdot 02 - \cdot 10 \cdot 00$
	<i>g</i>	314	$- \cdot 96 + \cdot 18 - \cdot 44$	$= - \cdot 013 + \cdot 19 + \cdot 03 + \cdot 11 + \cdot 02$

COMBINATION XX.

September 10, Evening; September 9 and 10, Morning.

Method of Observation B.

Mean Interval from $\delta + 4^{\text{h}} 76$.

Evening Observations, September 10.

	<i>e</i>	319	$+ \cdot 97 \Delta \alpha + \cdot 18 n - \cdot 21 z$	$= + \cdot 016 + \cdot 06 \Delta \delta + \cdot 02 m - \cdot 12 \kappa + \cdot 01 \kappa'$
lxiv.	<i>g</i>	320	$- \cdot 96 - \cdot 17 - \cdot 50$	$= + \cdot 028 + \cdot 20 + \cdot 03 + \cdot 12 + \cdot 03$
$\zeta_0 = 47^\circ$	<i>g</i>	321	$- \cdot 96 - \cdot 16 - \cdot 50$	$= + \cdot 093 + \cdot 20 + \cdot 03 + \cdot 13 + \cdot 03$
	<i>e</i>	322	$+ \cdot 97 + \cdot 16 - \cdot 21$	$= - \cdot 009 + \cdot 07 + \cdot 01 - \cdot 15 + \cdot 03$

Morning Observations, September 9.

	<i>g</i>	315	$- \cdot 96 \Delta \alpha + \cdot 19 n - \cdot 44 z'$	$= - \cdot 035 + \cdot 19 \Delta \delta + \cdot 04 m' - \cdot 55 \kappa - \cdot 11 \kappa'$
lxv.	<i>e</i>	316	$+ \cdot 98 - \cdot 20 - \cdot 27$	$= + \cdot 051 \cdot 00 + \cdot 03 + \cdot 55 \cdot 00$
$\zeta_0 = 62^\circ$	<i>g</i>	317	$- \cdot 96 + \cdot 20 - \cdot 44$	$= - \cdot 032 + \cdot 19 + \cdot 05 - \cdot 53 - \cdot 10$
	<i>e</i>	318	$+ \cdot 98 - \cdot 21 - \cdot 27$	$= + \cdot 053 \cdot 00 + \cdot 05 + \cdot 53 \cdot 00$

Morning Observations, September 10.

	<i>e</i>	333	$+ \cdot 97 \Delta \alpha - \cdot 19 n - \cdot 18 z$	$= + \cdot 096 + \cdot 10 \Delta \delta + \cdot 01 m' - \cdot 41 \kappa - \cdot 04 \kappa'$
lxvi.	<i>g</i>	334	$- \cdot 96 + \cdot 19 - \cdot 53$	$= + \cdot 019 + \cdot 20 + \cdot 04 + \cdot 41 - \cdot 09$
$\zeta_0 = 58^\circ$	<i>g</i>	335	$- \cdot 96 + \cdot 19 - \cdot 53$	$= + \cdot 040 + \cdot 20 + \cdot 05 + \cdot 42 - \cdot 09$
	<i>e</i>	336	$+ \cdot 97 - \cdot 20 - \cdot 18$	$= + \cdot 040 + \cdot 11 + \cdot 02 - \cdot 43 - \cdot 05$

TABLE I.—continued.

COMBINATION XXI.

September 11, Evening; September 10, Morning.

Method of Observation B.

Mean Interval from $\delta + 5^h 40$.

Evening Observations, September 11.

lxvii.	g 342	$-.96\Delta a$	$-.18n$	$-.59z$	$= +0.29 + .20\Delta\delta + .04m + .46\kappa + .10\kappa'$
$\zeta_0 = 50^\circ$	e 343	$+.94$	$+.17$	$-.12$	$= -0.02 + .26 + .01 - .46 + .13$

Morning Observations, September 10.

	e 329	$+.97\Delta a$	$-.14n$	$-.18z'$	$= +0.62 + .10\Delta\delta + .01m' + .25\kappa - .03\kappa'$
lxviii.	g 330	$-.96$	$+.15$	$-.53$	$= -0.32 + .20 + .03 - .24 - .05$
$\zeta_0 = 44^\circ$	g 331	$-.96$	$+.16$	$-.53$	$= +0.26 + .20 + .03 - .23 - .05$
	e 332	$+.97$	$-.18$	$-.18$	$= +0.42 + .10 + .01 + .22 - .02$

COMBINATION XXII.

September 19, Evening and Morning.

Method of Observation A and B.*

Mean Interval from $\delta + 13^h 98$.

Evening Observations.

lxix.	b 361, 365	$+.97\Delta a$	$+.03\Delta\delta$	$+.15n$	$-.62z''$	$= +0.96 + .03m + .10\kappa - .00\kappa'$
$\zeta_0 = 41^\circ$	e 362, 364	$-.96$	$-.20$	$-.14$	$-.57$	$= -1.32 + .03 - .11 - .02$
	e 363	$+.36$	$+.93$	$+.05$	$-.36$	$= -0.09 + .01 + .04 + .10$

Morning Observations.

lxx.	c 366	$+.33\Delta a$	$+.94\Delta\delta$	$-.04n$	$-.36z'''$	$= +0.17 + .01m' - .03\kappa - .08\kappa'$
$\zeta_0 = 31^\circ$	b 367	$+.97$	$+.03$	$-.11$	$-.60$	$= +1.81 + .02 - .10 - .00$
	e 368	$-.96$	$-.20$	$+.14$	$-.58$	$= -0.79 + .03 + .12 + .02$

COMBINATION XXIII.

September 24, Evening and Morning.

Method of Observation A and B.

Mean Interval from $\delta + 18^h 98$.

Evening Observations.

METHOD A.

lxxi.	c 380	$-.45\Delta a$	$+.89\Delta\delta$	$-.08n$	$-.39z$	$= -0.14 + .02m - .07\kappa + .15\kappa'$
$\zeta_0 = 56^\circ$	u 382	$+.87$	$-.45$	$+.15$	$-.39$	$= -1.25 + .01 + .13 - .06$

* In this combination the observations of the stars b and c were made by method A, but the star e was too faint for observation by this method, and method B was used. $\frac{1}{2}$ weight is given to the observation of bunch lxxi., in accordance with note, by combining 2 observations each of stars b and c .

TABLE I.—*continued.*

COMBINATION XXIII.—continued.

METHOD B.

lxxii.	β	384	$-.81\Delta\alpha - .56\Delta\delta - .10n - .54z'' = +0.67 + .02m - .08\kappa - .05\kappa'$
$\zeta_0 = 35^\circ$	b	385	$+ .97 + .10 + .11 - .29 = -1.22 + .01 + .11 + .01$

Morning Observations.

METHOD B.

lxxiii.	β	388	$-.82\Delta\alpha - .54\Delta\delta + .11n - .55z''' = +1.02 + .02m' + .08\kappa + .05\kappa'$
$\zeta_0 = 38^\circ$	b	389	$+ .97 + .11 - .13 - .28 = -1.53 + .01 - .11 - .01$

METHOD A.

lxxiv.	c	390	$-.49\Delta\alpha + .87\Delta\delta + .08n - .41z' = -.019 + .01m' + .06\kappa - .11\kappa'$
$\zeta_0 = 50^\circ$	a	391	$+ .86 - .47 - .15 - .37 = +0.01 + .03 - .13 + .07$

COMBINATION XXIV.

September 27, Evening; September 25, Morning.

Method of Observation A.

Mean Interval from $\mathcal{J} + 20^h.99$.

Evening Observations, September 27.

lxxv.	b	401	$+ .90\Delta\alpha + .37\Delta\delta + .15n - .16z = +0.22 + .01m - .71\kappa - .31\kappa'$
	β	402	$-.89 - .41 - .14 - .66 = +0.47 + .04 + .71 + .35$
	c	403	$-.65 + .75 - .10 - .51 = +0.13 + .02 + .56 - .65$
	a	404	$+ .79 - .59 + .12 - .25 = -.017 + .01 - .69 + .51$

Morning Observations.

lxxvi.	c	397	$-.57\Delta\alpha + .82\Delta\delta + .08n - .44z' = +0.10 + .02m' - .51\kappa + .73\kappa'$
	a	398	$+ .84 - .51 - .17 - .32 = +0.35 + .03 + .71 - .43$
	β	399	$-.85 - .49 + .16 - .60 = +0.79 + .06 - .71 - .41$
	b	400	$+ .96 + .18 - .19 - .22 = -.031 + .04 + .79 + .15$

COMBINATION XXV.

October 3, Evening and Morning.

Method of Observation, Distances and P. Angles A.

Mean Interval from $\mathcal{J} + 27^h.99$.

GREATER ZENITH DISTANCE.

Equations from the Measures of Distance.

Evening Observations.

lxxvii.	a	426	$-.96\Delta\delta - .04z = +0.27 - .29\Delta\alpha - .04n + .00m + .06\kappa - .18\kappa'$
$\zeta_0 = 49^\circ$	b	427	$+ .97 - .17 = +0.06 + .25 + .04 + .00 - .04 + .16$

TABLE I.—continued.

COMBINATION XXV.—continued.

Morning Observations.

lxxviii.	a	434	$-.99\Delta\delta - .03z' = +.001 - .08\Delta\alpha + .02n$	$.00m' - .01\kappa + .13\kappa'$
$\zeta_0 = 63^\circ$	b	435	$+.96 - .18 = -.063 + .28 - .06$	$.00 + .04 - .12$

Equations from the Measures of Position Angle.

Evening Observations.

lxxix.	a	426.p	$-8.01\Delta\alpha - 1.32n - k = -3.83 + 2.50\Delta\delta - .01$	$-1.52\kappa - .47\kappa'$
$\zeta_0 = 49^\circ$	b	427.p	$+1.86 + .025 - k = +3.51 - 0.50$	$.00 + .030 + .08$

Morning Observations.

lxxx.	a	434.p	$-10.87\Delta\alpha + 1.91n - k' = -5.95 + 0.94\Delta\delta$	$.00m + 1.42\kappa + .12\kappa'$
$\zeta_0 = 63^\circ$	b	435.p	$+1.75 - .032 - k' = +4.53 - 0.54$	$-.01 - .026 - .08$

LESSER ZENITH DISTANCE.

Equations from the Measures of Distance.

Evening Observations.

lxxxi.	b	428	$+0.97\Delta\delta - .17z'' = -.025 + .25\Delta\alpha + .04n$	$.00m - .03\kappa + .13\kappa'$
$\zeta_0 = 31^\circ$	a	429	$-.097 - .04 = +0.33 - .25 - .03$	$.00 + .03 - .11$

Morning Observations.

lxxxii.	a	430	$-.99\Delta\delta - .03z''' = -.042 - .15\Delta\alpha$	$.00n .00m' .00\kappa + .03\kappa'$
	b	431	$+0.96 - .18 = -.033 + .27 - .02$	$.00 + .01 - .05$
	b	432	$+0.96 - .18 = -.053 + .28 - .03$	$.00 + .02 - .06$
	a	433	$-.099 - .03 = -.017 - .09 + .02$	$.00 - .01 + .12$

Equations from the Measures of Position Angle.

Evening Observations.

lxxxiii.	b	428.p	$+1.85\Delta\alpha + 0.20n - k'' = +4.92 - 0.50\Delta\delta$	$.00m + .24\kappa + .07\kappa'$
$\zeta_0 = 31^\circ$	a	429.p	$-8.55 - .075 - k'' = -3.33 + 2.29$	$.00 - .94 - .25$

Morning Observations.

lxxxiv.	a	430.p	$-9.87\Delta\alpha + 0.85n - k''' = -5.15 + 1.59\Delta\delta$	$.00m' + .30\kappa + .05\kappa'$
	b	431.p	$+1.78 - .019 - k''' = +3.87 - 0.53$	$.00 - .09 - .03$
	b	432.p	$+1.78 - .021 - k''' = +2.80 - 0.53$	$.00 - .11 - .03$
	a	433.p	$-10.75 + 1.80 - k''' = -5.91 + 1.02$	$.00 + 1.28 + .12$

CHAPTER XV.

DEDUCTION OF FINAL RESULTS.

Having thus found the definitive combinations from which the parallax is to be deduced, the next step must be to treat the whole by such a system of reduction as will yield the most probable value of the parallax—in other words, to eliminate n in terms of m and m' from the whole of the equations by the method of least squares.

To do this I have proceeded by the following steps :—

- (1) Since the terms z or z' (which express the effect of an error in the scale-value at the time) are supposed different for each *bunch*, the first step must be to form normal equations from each *bunch* for each of the unknown quantities on the left hand side of the equations, and then eliminate z or z' from each set of normals, taking care to preserve the symmetry of the equations.
- (2) Since $\Delta\alpha$ and $\Delta\delta$ must be considered different for each combination (unless the tabular motion is supposed to be accurate), the next step was to form normal equations for $\Delta\alpha$, $\Delta\delta$, and n , by adding together all the normals of these names respectively in each combination, and so form the three definitive normals for each combination.
- (3) From these definitive normals for each combination, $\Delta\alpha$ and $\Delta\delta$ were then eliminated, taking care to preserve the symmetry of the equations, so bringing out from each combination an equation in n , in terms of m , m' , κ and κ' , in which the coefficient of n is the true weight of n .
- (4) From a preliminary solution, neglecting the errors due to erroneous tabular motion (*i.e.* neglecting κ and κ'), it was found that the final value of n could not be far from -0.25 , a result confirmed by the final solution. This value of n was therefore substituted in the normal equations for $\Delta\alpha$ and $\Delta\delta$ (where n must always have a very small coefficient), and thus two equations were obtained from each combination, containing on the left hand side only the unknown quantities $\Delta\alpha$ and $\Delta\delta$. In these equations the terms κ and κ' disappear, as the epoch of each combination is the mean of the epochs of the groups of which it is composed.

- (5) With the values of Δa and $\Delta \delta$ so found, the errors of the tabular motion are then determined (*i.e.* the values of κ and κ' for each combination), and the substitution of these values in the normals in n which result from each combination furnishes the definitive normal equations in n .

The sum of all the definitive normal equations in n is the final equation in n with the coefficient of its weight.

By steps 1, 2, and 3, the following normal equations in n are obtained.

TABLE I.

Combination.	Method of Observation.	Normals in n .	Tabular Corrections, where κ , κ' , Δa , and $\Delta \delta$ are peculiar to each combination.		
i.	A	$\cdot 113n = +\cdot 0127 + \cdot 002m + \cdot 008m'$	$+\cdot 099\kappa$	$+\cdot 032\kappa'$	
ii.	A	$\cdot 086 = -\cdot 153 \cdot 000 - \cdot 001$	$+\cdot 074$	$+\cdot 024$	
iii.	A	$\cdot 175 = +\cdot 032 \cdot 000 - \cdot 003$	$-\cdot 845$	$-\cdot 359$	
iv.	B	$\cdot 180 = +\cdot 069 + \cdot 001 - \cdot 002$	$-\cdot 109$	$-\cdot 347$	
v.	A	$\cdot 079 = -\cdot 004 \cdot 000 + \cdot 001$	$+\cdot 059$	$+\cdot 007$	
vi.	B	$\cdot 091 = +\cdot 098 + \cdot 001 - \cdot 002$	$+\cdot 075$	$-\cdot 006$	
vii.	B	$\cdot 109 = +\cdot 099 - \cdot 005 + \cdot 003$	$+\cdot 088$	$-\cdot 025$	
viii.	B	$\cdot 230 = +\cdot 140 \cdot 000 + \cdot 009$	$+\cdot 812$	$-\cdot 009$	
ix.	B	$\cdot 227 = -\cdot 074 \cdot 000 - \cdot 005$	$+\cdot 179$	$+\cdot 004$	
x. ¹	A	$\cdot 113 = +\cdot 053 - \cdot 002 + \cdot 001$	$+\cdot 087$	$-\cdot 023$	$\cdot 000\Delta a + \cdot 009\Delta \delta$
xi.	B	$\cdot 079 = -\cdot 304 + \cdot 001 - \cdot 001$	$+\cdot 056$	$+\cdot 022$	
xii. ²	A	$\cdot 038 = -\cdot 112 \cdot 000 \cdot 000$	$+\cdot 028$	$-\cdot 008$	$-\cdot 034\Delta a + \cdot 027\Delta \delta$
xiii. ⁵	B	$\cdot 547 = +\cdot 155 \cdot 000 \cdot 000$	$+\cdot 819$	$+\cdot 237$	
xiv. ³	B	$\cdot 057 = -\cdot 078 - \cdot 001 - \cdot 001$	$+\cdot 045$	$+\cdot 005$	$+\cdot 005\Delta \delta$
xv. ³	B	$\cdot 097 = -\cdot 192 - \cdot 001 \cdot 000$	$+\cdot 071$	$-\cdot 001$	$+\cdot 005$
xvi. ³	B	$\cdot 060 = +\cdot 021 \cdot 000 \cdot 000$	$+\cdot 046$	$+\cdot 005$	$\cdot 000$
xvii. ³	B	$\cdot 134 = +\cdot 009 - \cdot 006 - \cdot 001$	$+\cdot 103$	$+\cdot 003$	$\cdot 000$
xviii. ³	B	$\cdot 097 = -\cdot 112 - \cdot 001 \cdot 000$	$+\cdot 071$	$+\cdot 009$	$-\cdot 003$
xix. ³	B	$\cdot 145 = +\cdot 039 - \cdot 001 \cdot 000$	$+\cdot 111$	$+\cdot 008$	$-\cdot 010$
xx. ³	B	$\cdot 309 = -\cdot 493 + \cdot 001 - \cdot 004$	$-\cdot 208$	$-\cdot 010$	$+\cdot 048$
xxi. ³	B	$\cdot 106 = -\cdot 138 \cdot 000 \cdot 000$	$-\cdot 221$	$+\cdot 023$	$+\cdot 052$
xxii.	A & B	$\cdot 077 = -\cdot 034 - \cdot 001 + \cdot 002$	$+\cdot 061$	$+\cdot 013$	
xxiii.	A & B	$\cdot 109 = -\cdot 112 - \cdot 001 - \cdot 003$	$+\cdot 092$	$-\cdot 028$	
xxiv.	A	$\cdot 147 = +\cdot 062 \cdot 000 + \cdot 004$	$-\cdot 695$	$+\cdot 065$	
xxv. ⁴	P. Angle.	$\cdot 110 = +\cdot 134 \cdot 000 \cdot 000$	$+\cdot 097$	$+\cdot 022$	$-\cdot 063\Delta \delta$

Notes to Table I.

1, 2. These equations result from observations of two comparison stars only, but these stars are almost exactly on opposite sides of the planet. It is therefore necessary to retain $\Delta\alpha$ and $\Delta\delta$ on the right hand side of the equation, which is equivalent to assuming that the Tabular Errors will otherwise be determined sufficiently well to permit the accurate projection of the planet on the great circle joining the two stars.

3. In all these combinations the comparison stars were nearly of the same declination with the planet, and almost exactly on opposite sides of it. The resulting equations are therefore suitable for determining $\Delta\alpha$ and n , and only a very approximate knowledge of the value of $\Delta\delta$ is required to project the planet on the parallel of R.A. $\Delta\delta$, however, was determined for most of those dates by separate observations made either after the evening or before the morning observations for parallax.

4. In this combination, which depends on position angles, the comparison stars were nearly due N. and S. of the planet, and the resulting equations are therefore suitable for determining $\Delta\alpha$ and n . $\Delta\delta$ is determined by the simultaneous observations of distance, but the normal given in the table depends on the position angles only.

5. This equation contains on the right hand side the still further tabular corrections

$$-0.010z \qquad +0.104z',$$

the distance of *Mars* having been measured from one star only in this combination, the terms z and z' could not be eliminated.

Next by step 4 the following values $\Delta\alpha$ and $\Delta\delta$ are obtained.*

TABLE II.

Combination.	Method of Observation.	Interval from Opposition.	$\Delta\alpha$	$\Delta\delta$	Definitive Values of $\Delta\alpha$ and $\Delta\delta$ by Successive Approximation.	
i.	A	d -35.88	-0.21	+0.11	-0.21	+0.11
ii.	A	-30.92	- .34	+ .14	- .34	+ .14
iii.	A	-24.89	- .05	- .26	- .05	- .26
iv.	B	-22.15	- .14	- .39	- .14	- .39
v.	A	-20.93	- .48	- .69	- .48	- .69
vi.	B	-19.93	+ .25	- .48	+ .25	- .48
vii.	B	-16.98	+ .84	- .63	+ .84	- .63
viii.	B	-14.92	+ .90	- .64	+ .90	- .64
ix.	B	-11.91	+ .10	- .50	+ .10	- .50
x, xi.	B	- 6.96	- .06-0.07 $\Delta\delta$		- .02	

* The normal equations from which the values of $\Delta\alpha$ and $\Delta\delta$ are derived are given in the O. M.

TABLE II.—continued.

Combination.	Method of Observation.	Interval from Opposition.	$\Delta\alpha$	$\Delta\delta$	Definitive Values of $\Delta\alpha$ $\Delta\delta$ by Successive Approximation.
Sept. 2	A	^d — 3.03	"	— .70 + 0.09 $\Delta\alpha$	" — .65
4	B	— 1.14		— .99 + .75	— .54
4	A	— 1.08		— .98 + .67	— .58
xii.	Two stars only observed differing nearly 180° in P. Angle.				
xiii.	B	+ 0.09	+ .73	— .71	+ .73 — .71
xiv.	B	+ 1.01	+ .72 — 0.11 $\Delta\delta$		+ .80
xv.	B	+ 1.04	+ .67 + .03		+ .65
xvi.	B	+ 2.01	+ .37 — .15		+ .47
xvii.	B	+ 2.97	+ .13 — .04		+ .16
xviii.	B	+ 3.07	— .28 — .14		— .18
Sept. 8	B	+ 3.10		— 0.75 — .15	— .71
xix.	B	+ 4.07	+ .36 — .08		+ .42
Sept. 9	B	+ 4.10		— 1.00 + .64	— .80
xx.	B	+ 4.76	+ .24 + .02	.	+ .23
Sept. 10	B	+ 4.94		— 0.70	— .70
xxi.	B	+ 5.40	+ .26 + .07		+ .21
xxii.*	A & B	+ 13.98	+ 1.33	— 0.52	[+ 1.33] [— .52]
xxiii.*	A & B	+ 18.98	— 1.03	— .63	[— 1.03] [— .63]
xxiv.	A	+ 20.99	— .17	— .21	— .17 — .21
xxv.	A	+ 27.99		+ .09	+ .09
xxv.	P. Angles	+ 27.99	+ .74		+ .74

Running the eye over these values of $\Delta\alpha$ and $\Delta\delta$, the reader will be struck with the markedly even manner in which the values of $\Delta\delta$ run as compared with the values of $\Delta\alpha$. This is the more remarkable, since the arrangement of the comparison stars was made with a view to determine the Right Ascensions rigidly, and the determinations of R.A. therefore come out with much greater numerical weight than the Declinations.

* Combinations xxii. and xxiii. are not suitable for determining absolute places, because the stars on opposite sides of the planet were observed by different methods. These combinations were similar, however, evening and morning, and are therefore suitable for parallax determination.

The following appear to be possible causes for the greater apparent errors of the Right Ascensions :—

- (1) The star-places may be much less accurately determined in R.A. than in Declination.
- (2) The Heliometer measures of distance may be more accurately made along directions near Position Angle 0° and 180° than at right angles to these directions.
- (3) There may be real tabular errors that cannot be developed in powers of t and t^2 .

With regard to the *first* of these possible causes of error, I have carefully examined the question, and specially with reference to the corrections for magnitude determined in Chapter IX., and I find that the discordances in question are considerably greater if these corrections are omitted. Still, I have little doubt that the star-places are better determined in Declination than in Right Ascension.

As to the *second* possible cause of apparent error, I think it is most probable that measure of distance along Position Angles near 0° or 180° will almost certainly be made with more accuracy than in the direction 90° or 270° , because in the prime vertical, and in a low latitude like Ascension, the limbs of the planet will not, in the former case, be sensibly affected with atmospheric chromatic dispersion, but will be so in the latter case.

As to the *third* possible cause of apparent error, though I am unable to foresee any real perturbation of short period such as is apparent from the values of $\Delta\alpha$, yet I can conceive it to be possible that *apparent* periodic errors may be caused by changes in the appearance of the limb, according as a bright or dark district of the planet happens to come to the limb.

If there is no reality in the periodic variation of $\Delta\alpha$, then we can expand $\Delta\alpha$ and $\Delta\delta$ in terms of the time from opposition, and I find the value of $\Delta\alpha$ may be expressed as follows, if we omit the representation of the value of $\Delta\alpha$ resulting from combinations x. and xi. and xxv.,

$$\Delta\alpha = +0''.44 - 0.0114t - 0.00112t^2 \quad (1)$$

where t is the time from opposition reckoned in days as given in the preceding Table II.

Assuming, on the other hand, the possible reality (due to some un-

ascertained cause) of an error that can be expressed in terms of the sine and cosine of an unknown period u , then the expression for $\Delta\alpha$ becomes, as representing all the observations,

$$\left. \begin{aligned} \Delta\alpha = & +0''.43 & \pm 0''.07 \\ & +0.0035t & \pm 0.0032 \\ & -0.000367t^2 & \pm 0.00016 \\ & + \left(\begin{smallmatrix} 0''.376 \\ \pm 0''.071 \end{smallmatrix} \right) \times \sin(u + 122^\circ.3 \pm 12^\circ.6), \\ \text{where } u = & \frac{t}{13^d.626 \pm 0.067} \times 360^\circ, \end{aligned} \right\} \quad (2)$$

By Equation (1) we obtain

$$\frac{[vv]}{n-1} = 0.1034,$$

or the probable error of one determination of R.A.

$$= \pm 0''.217,$$

but this excludes three of the equations.

By Equation (2), and representing all the observations, we get

—	Number of Unknown Quantities.	Σvv	Corresponding Probable Error of One Determination.
Supposing only a constant error	1	3.56	$\pm 0''.285$
„ terms depending on t and t^2 only	3	2.69	$\pm 0''.261$
„ including periodic terms also ...	6	1.30	$\pm 0''.199$

The Declinations are very accurately represented by the expression

$$\Delta\delta = -0''.66 + 0.0059t + 0.00077t^2 \quad (3)$$

the comparison of which with the observations gives

$$\frac{[vv]}{n-1} = 0.0071,$$

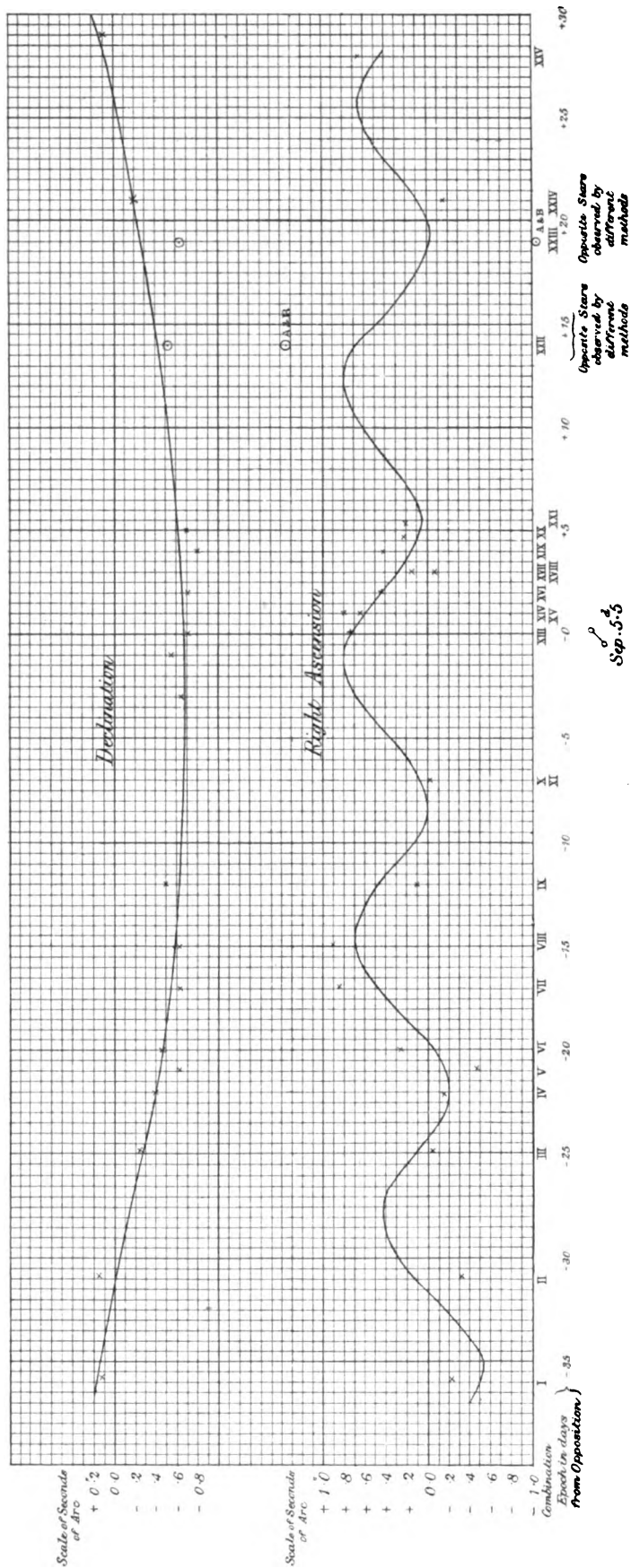
and the probable error of one determination of Declination

$$= \pm 0''.057.$$

The representation of these expressions is given in the following Table III.

TABLE III.

—	Observations compared with equation (1), i.e. if no error of short period is assumed.	Observations compared with equation (2), i.e. assum- ing possibility of errors of short period.	Observations compared with equation (3).
	$\Delta\alpha$ O—C	$\Delta\alpha$ O—C	$\Delta\delta$ O—C
i.	+ 0'39	+ 0'33	— 0'01
ii.	— '05	— '52	+ '04
iii.	— '07	— '10	— '07
iv.	— '27	+ '06	— '02
v.	— '66	— '33	+ '25
vi.	+ '04	+ '29	'00
vii.	+ '53	+ '35	— '09
viii.	+ '54	+ '23	— '06
ix.	— '31	— '30	+ '12
x., xi.	[— '52]	— '10	
Sept. 2			+ '02
4			+ '13
4			+ '09
xiii.	+ '29	— '01	— '05
xiv.	+ '37	+ '18	
xv.	+ '22	+ '03	
xvi.	+ '05	'00	
xvii.	— '24	— '15	
xviii.	— '58	— '48	
Sept. 8			— '05
xix.	+ '04	+ '28	
Sept. 9			— '17
xx.	— '13	+ '14	
Sept. 10			— '09
xxi.	— '14	+ '14	
xxii.			
xxiii.			
xxiv.	+ '11	— '23	— '01
xxv.			— '02
xxv. P. Ang.	[+ 1'80]	+ '13	



These results are, perhaps, better shown graphically on the accompanying plate.

It is difficult to understand why the Declinations should be so beautifully represented as compared with the Right Ascensions. I have already mentioned what I believe to be the combined reasons for the difference.

It is worthy of remark that the period in R.A. (13.626 days) given in Equation (2) is curiously close to that of a term depending on twice the Moon's mean longitude, but I confess I can find no cause for such a perturbation.

It fortunately happens, as will afterwards appear, that so far as the resulting parallax is concerned, it is immaterial which of the two methods of correcting the tabular motion in the Right Ascension is adopted.

The following Table IV. gives the values of κ and κ' for each combination, according to both systems.

TABLE IV.

Corrections to the Daily Tabular Motion of Mars in R.A. and Dec.

For Combination.	Without Periodic Errors. κ	With Periodic Errors. κ	κ'
i.	+ 0.06	- 0.04	- 0.03
ii.	+ .05	+ .20	- .03
iii.	+ .05	- .14	- .03
iv.	+ .05	.00	- .03
v.	+ .04	+ .08	- .03
vi.	+ .04	+ .14	- .03
vii.	+ .03	+ .15	- .03
viii.	+ .02	+ .03	- .02
ix.	+ .01	- .14	- .01
x.	.00	+ .09	.00
xi.	.00	+ .09	.00
xii.	.00		.00
xiii.	- .01	- .15	+ .01
xiv.	- .01	- .15	+ .01

TABLE IV.—*continued.*

For Combination.	Without Periodic Errors. κ	With Periodic Errors. κ	κ'
xv.	— '01	— '15	+ '01
xvi.	— '01	— '15	+ '01
xvii.	— '01	— '17	+ '01
xviii.	— '01	— '17	+ '01
xix.	— '02	— '15	+ '02
xx.	— '03	— '08	+ '02
xxi.	— '03	— '02	+ '02
xxii.	— '04	— '12	+ '03
xxiii.	— '05	— '04	+ '03
xxiv.	— '05	+ '12	+ '03
xxv.	— '10	— '15	+ '04

Substituting these values of κ and κ' in the normal equations in n of Table I. and also the numerical equivalents of Δa and $\Delta \delta$ as computed by our equations (1), (2), and (3), the following definitive normal equations in n are obtained, in which the terms m , m' , z , z' , are, for the present, neglected.

There are, however, three of the equations in n that should not be included in a general discussion, viz.,

XII. From the very unfavourable observations under which the observations were made (see O.M.)

XI. and VIII. Because one of the stars of comparison (λ) was very faint, and could only with excessive strain of the observer's eye be compared with the planet. (See foot-note, page 39.)

Omitting these equations, therefore, the definitive normals in n are given in the following Table V

TABLE V.
Definitive Normal Equations in n .

Combination.	Method of Observation.	Corrected for Tab. Errors without Periodic Errors, i.e., by (1) and (3).	Corrected for Tab. Errors supposing Periodic Errors, i.e., by (2) and (3).
i.	A	$\cdot 113n = +0''132$	or $= +0''122$
ii.	A	$\cdot 086 - \cdot 150$	$- \cdot 139$
iii.	A	$\cdot 175 + \cdot 001$	$+ \cdot 161$
iv.	B	$\cdot 180 + \cdot 074$	$+ \cdot 079$
v.	A	$\cdot 079 - \cdot 002$	$+ \cdot 001$
vi.	B	$\cdot 091 + \cdot 101$	$+ \cdot 108$
vii.	B	$\cdot 109 + \cdot 103$	$+ \cdot 113$
ix.	B	$\cdot 227 - \cdot 072$	$- \cdot 099$
x.	A	$\cdot 113 + \cdot 048$	$+ \cdot 056$
xiii.	B	$\cdot 547 + \cdot 149$	$+ \cdot 034$
xiv.	B	$\cdot 057 - \cdot 082$	$- \cdot 089$
xv.	B	$\cdot 097 - \cdot 197$	$- \cdot 207$
xvi.	B	$\cdot 060 + \cdot 021$	$+ \cdot 014$
xvii.	B	$\cdot 134 + \cdot 008$	$- \cdot 009$
xviii.	B	$\cdot 097 - \cdot 115$	$- \cdot 126$
xix.	B	$\cdot 145 + \cdot 045$	$+ \cdot 030$
xx.	B	$\cdot 309 - \cdot 523$	$- \cdot 512$
xxi.	B	$\cdot 106 - \cdot 166$	$- \cdot 169$
xxii.	A & B	$\cdot 077 - \cdot 036$	$- \cdot 041$
xxiii.	A & B	$\cdot 109 - \cdot 118$	$- \cdot 117$
xxiv.	A	$\cdot 147 + \cdot 098$	$- \cdot 020$
xxv.	Pos. Angle.	$\cdot 110 + \cdot 019$	$+ \cdot 014$

These furnish the normal equations

$$3\cdot 168n = - \cdot 662 \quad \text{or} = - \cdot 796$$

$$\text{and} \quad n = - \cdot 209 \quad \quad \quad = - \cdot 251$$

The substitution of these values gives,

$$\frac{[p \cdot vv]}{n-1} = \cdot 1204 \quad \text{or} = \cdot 1280$$

Whence P. E. of 1 observation $\dots = \pm 0''234$ or $= \pm 0''241$

And the resulting values of the parallax $= 8\cdot 782 \pm 0\cdot 016$ or $= 8\cdot 778 \pm 0\cdot 019$

The arithmetical means of the separate results, without weights, are $\dots = 8\cdot 777$ or $= 8\cdot 775$

The weights employed in the above investigation are the mathematical representations of the supposition that each observation is equally good. It is possible, however, *à priori*, that the result of each combination may be affected by some constant error, such, for example, as a difference from normal brilliancy at the point of the limb which is observed. Such an error would not be diminished by increasing the number of observations. In order to test this I have formed from each combination an equation of the form

$$\frac{r^2}{p} + s^2 = vv,$$

where r is the accidental error, p the weight, vv the square of the residuals, and s the systematic error sought.

This equation multiplied by p gives

$$r^2 + ps^2 = p \cdot vv,$$

which by least squares leads to the normal equations derived from m equations—

$$\begin{aligned} mr^2 + [p]s^2 &= [p \cdot vv] \\ [p]r^2 + [pp]s^2 &= [pp \cdot vv], \end{aligned}$$

whence the normals

$$\begin{aligned} 22.00 \ r^2 + 3.168s^2 &= 2.688 \\ 3.168 \ r^2 + .695 \ s^2 &= 0.272, \end{aligned}$$

which lead to a negative value of s^2 , or in other words show that the weights of the higher weighted equation have not been too great, but rather the reverse.

The explanation of this appears to be that there is very little systematic error attaching to the work of a particular evening, but when numerous observations were secured, and therefore combinations of high weight were obtained, the atmospheric conditions were, as a rule, more favourable for accurate observation than on those nights when the observations were interrupted by clouds, and when consequently few observations were secured.

I therefore believe that we are entitled to conclude the $\odot$'s parallax resulting from this investigation to be

$$8''.78 \pm 0''.012.$$

It is obvious that the terms in m , m' , z and z' , have no sensible influence whatever upon the parallax. Their actual coefficients in determining $\Delta\pi$ come out

$$-0.0032m + 0.0018m' - 0.003z + 0.0026z'.$$

Thus even if we put m and $m' = -100$, i.e. if the corrections for refraction were *entirely neglected*, the final result for the parallax would only be changed $+0''.014$; and, as in our present knowledge of refraction it is impossible that BESSEL'S tables can be $\frac{1}{100}$ part in error, it is impossible that an error equal to $\pm 0''.00014$ can be due to ordinary errors of refraction.

With regard to the terms z and z' , the extreme values found for these terms in the discussion of the star-places were (see Table II., Chapter IX., O.M.)

$$\text{for } z + 2''.3 \text{ and } -0''.35 \quad \text{for } z' + 1''.5 \text{ and } -0''.97,$$

and these extreme values occur only in exceptional circumstances. It is obvious that even these opposite extreme cases (which never occur on the same night) cannot sensibly affect the resulting parallax.

In order, however, to obtain a crucial test of the general symmetry of the arrangement of the comparison stars and the consequent freedom of the result from systematic errors due to errors in the adopted mean scale-value, or to changes in the actual scale-value from time to time, I have discussed the whole of the observations by the following simple process.

The equation resulting from the evening observations is subtracted from the equation resulting from the morning observations of the same star, or *vice versa*. Thus, for example,

Star of Comparison.	1877, July 31. COMBINATION I.							
q	{ 42-43	$.07n = -0''.29 + .70z - .70z' + .05\kappa + .29\kappa' + .01\Delta\alpha$	$.00\Delta\delta$					
	{ 47-48	$.05 = - .11 + .70 - .69 + .03 + .15$	$.00$	$.00$				
n	{ 43-52	$.28 = + .54 + .44 - .44 + .26 + .03$	$.00$	$-.01$				
	{ 46-49	$.20 = - .73 + .44 - .44 + .17 + .02$	$.00$	$-.01$				
t	{ 51-44	$.24 = + .75 - .42 + .42 + .21 + .08$	$.00$	$+.01$				
	{ 50-45	$.22 = + .21 - .42 + .42 + .19 + .09$	$.00$	$+.01$				

from which by least squares a single normal equation in n is obtained.

In this way the following normals are derived for each combination.

TABLE VI.

Combination.	Tabular Corrections peculiar to each Combination.							
i.	$\cdot 232n = +\cdot 206 + \cdot 102z - \cdot 102z'$	+	$\cdot 204\kappa$	+	$\cdot 079\kappa'$	+	$\cdot 001\Delta\alpha$	$\cdot 000\Delta\delta$
ii.	$\cdot 155 = -\cdot 159 + \cdot 147 - \cdot 147$	+	$\cdot 137$	+	$\cdot 052$	-	$\cdot 002$	$\cdot 000$
iii.	$\cdot 374 = -\cdot 018 - \cdot 160 - \cdot 217$	-	$\cdot 1803$	-	$\cdot 717$	+	$\cdot 025$	$\cdot 057$
iv.	$\cdot 360 = +\cdot 146 + \cdot 014 - \cdot 018$	-	$\cdot 217$	-	$\cdot 086$	+	$\cdot 005$	$\cdot 006$
v.	$\cdot 122 = +\cdot 007 + \cdot 043 - \cdot 041$	+	$\cdot 091$	+	$\cdot 008$	-	$\cdot 005$	$\cdot 010$
vi.	$\cdot 133 = +\cdot 138 - \cdot 005 + \cdot 012$	+	$\cdot 111$	-	$\cdot 005$	-	$\cdot 007$	$\cdot 011$
vii.	$\cdot 168 = +\cdot 190 - \cdot 026 + \cdot 032$	+	$\cdot 133$	-	$\cdot 042$	-	$\cdot 004$	$\cdot 011$
viii.	$\cdot 461 = +\cdot 253 + \cdot 138 - \cdot 032$	+	$\cdot 660$	-	$\cdot 019$	+	$\cdot 002$	$\cdot 054$
ix.	$\cdot 459 = -\cdot 130 - \cdot 082 + \cdot 114$	+	$\cdot 362$	+	$\cdot 021$	+	$\cdot 016$	$\cdot 004$
x.	$\cdot 231 = +\cdot 068 + \cdot 061 - \cdot 041$	+	$\cdot 177$	-	$\cdot 044$	+	$\cdot 003$	$\cdot 003$
xi.	$\cdot 159 = -\cdot 633 - \cdot 026 + \cdot 037$	+	$\cdot 114$	+	$\cdot 040$	+	$\cdot 015$	$\cdot 006$
xii.	$\cdot 077 = -\cdot 209 - \cdot 020 - \cdot 020$	+	$\cdot 057$	-	$\cdot 015$	$\cdot 000$	$\cdot 001$	
xiii.*	$\cdot 1094 = +\cdot 310 - \cdot 020 + \cdot 208$	+	$\cdot 638$	+	$\cdot 474$	-	$\cdot 022$	$\cdot 014$
xiv.	$\cdot 175 = -\cdot 158 + \cdot 152 - \cdot 140$	+	$\cdot 136$	+	$\cdot 015$	$\cdot 000$	$\cdot 005$	
xv.	$\cdot 205 = -\cdot 452 + \cdot 077 - \cdot 061$	+	$\cdot 151$	-	$\cdot 003$	$\cdot 000$	$\cdot 004$	
xvi.	$\cdot 141 = +\cdot 090 + \cdot 078 - \cdot 067$	+	$\cdot 106$	+	$\cdot 011$	$\cdot 000$	$\cdot 000$	
xvii.	$\cdot 212 = +\cdot 075 - \cdot 046 + \cdot 066$	+	$\cdot 160$	+	$\cdot 003$	$\cdot 000$	$\cdot 000$	
xviii.	$\cdot 192 = -\cdot 231 + \cdot 021 - \cdot 005$	+	$\cdot 143$	+	$\cdot 019$	$\cdot 000$	$\cdot 003$	
xix.	$\cdot 225 = +\cdot 129 - \cdot 033 + \cdot 047$	+	$\cdot 174$	+	$\cdot 013$	$\cdot 000$	$\cdot 010$	
xx.	$\cdot 541 = -\cdot 888 - \cdot 202 + \cdot 188$	-	$\cdot 282$	-	$\cdot 060$	+	$\cdot 004$	$\cdot 007$
xxi.	$\cdot 255 = -\cdot 287 - \cdot 161 + \cdot 121$	-	$\cdot 466$	-	$\cdot 002$	+	$\cdot 010$	$\cdot 053$
xxii.	$\cdot 154 = -\cdot 096 + \cdot 034 - \cdot 026$	+	$\cdot 123$	+	$\cdot 027$	-	$\cdot 003$	$\cdot 001$
xxiii. A	$\cdot 116 = -\cdot 386 + \cdot 055 - \cdot 045$	+	$\cdot 099$	-	$\cdot 081$	+	$\cdot 003$	$\cdot 003$
xxiii. B	$\cdot 101 = +\cdot 148 - \cdot 044 + \cdot 048$	+	$\cdot 086$	+	$\cdot 026$	+	$\cdot 002$	$\cdot 002$
xxiv.	$\cdot 322 = +\cdot 120 - \cdot 163 + \cdot 092$	-	$\cdot 535$	+	$\cdot 137$	+	$\cdot 009$	$\cdot 030$

* The factor of n (in the process of reduction necessary in the case of combination xiii.) is obtained with its true weight for a single equation resulting from a single group, or measure. But all the other factors of n are derived from the arithmetical sum of the evening and morning factors. Thus if f''' and $f_{///}$ are the factors of n in an equation resulting from an evening and from a morning observation respectively, we should have, in the case of combination xiii., the combining factor $= f'''^2 + f_{///}^2$, and in all the other equations, except xxv., it would be $(f''' + f_{///})^2$. If then f''' and $f_{///}$ are equal (which is generally the case in observations near opposition), the combining factor in case of combination xiii. becomes $2f^2$, and in all other combinations (excepting xxv.) $= 4f^2$. Thus to equalise the weights of combinations xiii. and xxv., the resulting normals must be multiplied by two.

Substituting in these equations the previously determined values of κ , κ' , $\Delta\alpha$ and $\Delta\delta$, and taking the sum, we obtain

$$6.664n = -1.749 + .409z - .709z',$$

to which, for sake of comparison, must be added the equation (xxv.) from the position angles, multiplied (like xiii.) by 2, viz.,

$$.220n = +.02.$$

the sum giving

$$\begin{aligned} 6.884n &= -1.729 + .409z - .709z' \\ n &= -.025 + .06z - .11z'. \end{aligned}$$

In order to determine as exactly as possible the most probable values of z , i.e. the exact error of the adopted screw-value in 10,000'', I have compared all the observed distances of the comparison stars with their tabular distances, after the latter have been corrected for errors in the R.A. due to magnitude. In this way I have obtained from forty-five pairs of stars observed East of the meridian, in circumstances of altitude and temperature exactly similar with the *Mars* observations, the normal equation

$$14.76z = +8.78,$$

and from thirty-six pairs of stars observed West of the meridian in similar circumstances

$$13.94z' = +3.94,$$

whence

$$z = +0.60 \text{ and } z' = +0.28.$$

The difference of these small quantities ($0''.32$) is the amount which the screw-value over 10,000'' in the evening exceeds the screw-value in the morning observations. After a most careful study of the whole matter I find that this difference is found chiefly in the observations made soon after sunset, and is due to the different rate of cooling of the object-glass and the tube. It was my practice to begin work as soon after sunset as possible, hence the

difference in question. A few hours after sunset these differences entirely disappear, or at least are not systematic.

But, as will be seen from the equations, such errors produce no sensible influence whatever upon the resulting parallax, for we have for the combined influence of z and z' on the value of n only the numerical equivalent $0''.005$, a quantity which is equivalent to a change in the resulting parallax of $+0''.0004$.

From the measures of distance only we find the value of

$$[p \cdot vv] = 8.731,$$

whence the probable error of n

$$= \pm 0.16,$$

and the value of

$$\Delta\pi = -0.022 \pm 0.014,$$

or rejecting the doubtful combinations viii. xi. and xii., we get

$$[p \cdot vv] = 5.793,$$

and

$$\Delta\pi = -0.016 \pm 0.013,$$

or

$$\pi = 8.784 \pm 0.013.$$

Thus we obtain the mean solar parallax practically of identical value and practically of equal accuracy by this simple method of reduction with that obtained from the laborious method previously employed.

This, of course, arises from the perfectly symmetrical general arrangement of the stars of comparison; for if the probable error of the parallax is computed from the separate equations instead of the combinations (by which the scale value is eliminated) a very much larger probable error will be found.

The following table gives the separate results of the various combinations from which the definitive value of the parallax has been deduced.

TABLE VII.
Separate Results for Mean Solar Parallax.

Combination.	Date 1877.	Method of Observation.	When scale value is determined from each bunch of observations.			When scale value is assumed constant and known.		
			Relative Weight.	π	Difference from Mean.	Relative Weight.	π	Difference from Mean.
i.	July 31, E & M	A	·113	8 ⁹⁰	+0 ¹²	·232	8 ⁸⁸	+0 ¹⁰
ii.	Aug. 5, E & M	A	·086	·65	— ·13	·155	·71	— ·07
iii.	{ 12, E 10, M }	A	·175	·80	+ ·02	·374	·78	·00
iv.	{ 14, E 13, 14 M }	B	·180	·83	+ ·05	·360	·83	+ ·05
v.	15, E & M	A	·079	·80	+ ·02	·122	·81	+ ·03
vi.	16, E & M	B	·091	·90	+ ·12	·133	·90	+ ·12
vii.	19, E & M	B	·109	·88	+ ·10	·168	·90	+ ·12
ix.	24, E & M	B	·227	·77	— ·01	·459	·78	·00
x.	29, E & M	A	·113	·84	+ ·06	·231	·83	+ ·05
xiii.	Sept. 4-6, E & M	B	·547	·82	+ ·04	1·094	·82	+ ·04
xiv.	6, E & M	B	·057	·67	— ·11	·175	·72	— ·06
xv.	6, E & M	B	·097	·62	— ·16	·205	·60	— ·18
xvi.	7, E & M	B	·060	·83	+ ·05	·141	·85	+ ·07
xvii.	8, E & M	B	·134	·81	+ ·03	·212	·83	+ ·05
xviii.	8, E & M	B	·097	·70	— ·08	·192	·69	— ·09
xix.	9, E & M	B	·145	·83	+ ·05	·225	·85	+ ·07
xx.	{ 10, E 9, 10, M }	B	·309	·65	— ·13	·541	·66	— ·12
xxi.	{ 11, E 10, M }	B	·106	·66	— ·12	·255	·70	— ·08
xxii.	19, E & M	A & B	·077	·76	— ·02	·154	·74	— ·04
xxiii.	24, E & M	A & B	·109	·71	— ·07	·217	·70	— ·08
xxiv.	{ 27, E 25, M }	A	·147	86	+ ·08	·322	·86	+ ·08
xxv.	Oct. 3, E & M	P. Angles	·110	·81	+ ·03			

Besides these two solutions, I have given in the *Monthly Notices* (vol. xxxix. p. 437) the result of another solution in which the places of the stars of comparison are regarded as absolutely known. This method, however, is liable to the disadvantage that it involves, to some extent, the errors of the places of the comparison stars, and the uncertainty of the errors of the tabular motion over much longer periods than in the strictly limited combinations employed in the previous discussions.

I believe that the latter is by far the more important consideration; at least, I am not able to account for the much greater probable error of a single observation by the former hypotheses.

When all the observations were formed into thirteen combinations, and the thirteen final normal equations in n were cleared from the terms $a, b, c, a', b',$ and c' (which represent the errors of the tables developed in terms of the time from Opposition and the distance from the Earth), I found that

$$[p \cdot vv] = 12.036,$$

and the weight of n

$$= 5.202.$$

Hence the probable error of the resulting parallax was

$$\pm 0.026,$$

and the probable error of a single measure of distance was

$$\pm 0.676.$$

By the method of symmetrical evening and morning observations, employed in the previous discussion, in which the errors of the star-places are eliminated from the equations in n , the probable error of one observation of distance was found to be

$$\pm 0.236.$$

Hence the probable error of a star's place from the meridian observations, if the development of $\Delta\alpha$ and $\Delta\delta$ could be accurately made in terms of Δ, t , and t^2 , would be

$$\sqrt{.676^2 - .236^2} = \pm .633,$$

a quantity which I consider to be very improbable, and which I therefore think more likely to be partly accounted for by some real or apparent tabular error of short period such as that indicated in the preceding discussion.

It has already been shown that such an error, whether real or apparent, produces no sensible influence on the accuracy or value of the resulting parallax, when the intervals between the epochs of opposite parallax in R.A. are so short as those employed in the combinations of the preceding discussions of symmetrical observations. I have therefore not thought it necessary to print details of the discussion in which the star-places are regarded as absolutely known, since, with the evidence just quoted, I think the reader will agree in the choice I have made as to the best method of definitive reduction. I propose, however, to deposit with the Society details in MSS. for preservation with the O.M. of the present volume.

The following Table contains a summary of all the results obtained by these discussions.

Mean Solar Parallax.

RESULTS, when only symmetrical evening and morning observations are employed, therefore not affected by errors of the star-places.

Scale Value supposed unknown, and deduced from each Bunch of Observations.

			Arithmetical Mean without weight.
From the symmetrical observations combined		8.777 ± 0.013	"
„ the same rejecting doubtful combinations viii. xi. xii.		8.782 ± 0.012	8.777
„ „ on the supposition of periodic variation in the R.A.		8.778 ± 0.012	8.775

Scale Value supposed known, and constant.

From all the symmetrical observations of distance		8.778 ± 0.014
„ the same rejecting the doubtful combinations viii. xi. xii.		8.784 ± 0.013

RESULTS, from all the observations, whether symmetrical or otherwise, made by whatever method, the places of the stars being regarded as absolutely determined.

From all the observations of distance combined		8.783 ± 0.026
„ observations of Greater Z.D. only		8.786 ± 0.034
„ „ Lesser Z.D. only		8.780 ± 0.040
„ „ before opposition		8.791 ± 0.034
„ „ after opposition		8.772 ± 0.040

Thus it appears that whatever method of reduction is employed, the resulting parallax is practically the same, viz.

8".78.

It remains to be considered what possible systematic errors may remain in the result.

Of these errors only one is in any degree evident from the agreement of the various values of the parallaxes deduced, viz., a small systematic difference in the results obtained before and after opposition.

From all the symmetrical observations we have

Mean $\odot$ ' parallax from all observations before opposition	8".80
" " after opposition	8".76

quantities which differ by more than they should according to their probable errors.

But in the observations where the star-places are considered known, and where a more continuous series of observations is available, this discordance is much less, viz.

Mean $\odot$ ' parallax from observations before opposition ...	8".79
" " after opposition ...	8".77

quantities which agree within the probable error of each.

The only cause to which it seems these differences (if systematic) can be attributed, is the effect of phase.

Meridian observations appear to indicate some real systematic difference of this kind. The discordances of the results for parallax determined from observations before and after opposition seem too large and too persistent in sign to be accounted for by chance. The most probable explanation appears to be, that observers estimate the contact of a web with a limb affected by phase differently according as the defective limb is the upper or lower one. Whatever error there may be due to such a cause, that error will, in the case of meridian observations, enter with its full amount into the measured angle of displacement of the planet, and will affect the deduced parallax with opposite signs before and after opposition, and it may be to something of

the kind that the following apparently systematic discordances in the results of the meridian observations of *Mars* are due.

Opposition of Mars, 1862.

	Before Opposition.	After Opposition.	Difference. Before — After.
	"	"	"
STONE, from Greenwich Observations combined with Cape of Good Hope and Williamstown (<i>Mem. R.A.S.</i> vol. xxiii. p. 183).	$\pi = 9.003$	8.827	+ 0.176
NEWCOMB, from combination of observations at Williamstown, Pulkowa, Helsingfors, Cape, Leiden, Greenwich, Santiago, Albany, Wash- ington (Appendix to Washington Observations, 1865).	$\pi = 8.875$	8.796	+ 0.079

Opposition of Mars, 1877.

DOWNING, from Leiden and Melbourne Observa- tions (<i>Astr. Nach.</i> 2288).	$\pi = 8.939$	8.883	+ 0.056
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But if the difference in the value of the parallax obtained from my heliometer observations before and after opposition is real, it cannot be thus accounted for ; because, by means of the reversing prism, the position of the defective limb with respect to the observer's eye is reversed in each alternate bisection. Besides this, the slightly smaller value of the parallax after opposition results entirely from observations made during the period September (4—11).

Now, during this period, not only was the weather, as regards definition, much the finest experienced during the whole series, and the stars of comparison the most favourable possible, but also the limbs of the planet at the points of contact with the comparison stars were, during that period, *absolutely free from phase*.

In my report of the observations sent from Ascension to the Society, I have called attention to the peculiar value of this series (*Monthly Notices*, xxxviii. p. 9). The parallax which results from this series alone is

$$8''.75,$$

the observations before and after this series giving larger values.

If the heliometer observations are affected by phase, the observations preceding and following this series should give for one a larger and for the

other a smaller value of the parallax, but I can conceive no way in which (when the mode of observation is considered) the tendency of phase would be to increase the observed displacement of the planet both if the defective limb is preceding or following. Thus it seems reasonable to conclude that the small discordance in question is probably accidental.

If we regard the fact of the exceptionally favourable circumstances in which the observations during the period September (4-11) were made, it might be desirable to accord a somewhat higher weight to that series than to the other observations. But as such an assumption must be purely arbitrary, and as throughout the observations and their discussion I endeavoured only to make observations under sufficiently satisfactory conditions, and to attach to all such observations an equal weight, I see no good reason now for departing from such a wholesome rule. I therefore merely indicate the possibility^{*} that, on the grounds quoted, the more probable value may be as near to $8''.77$ as $8''.78$.

It is worthy of remark, however, that in the only existing determination of the $\odot$ ' parallax from a minor planet, in which the observations extend for some time before as well as after opposition, a like discordance is met with, viz., in Dr. Galle's discussion of the extra-meridian observations of *Flora* at opposition in October and November 1873.*

Dr. Galle divides the discussion into three series, which give the following results :—

27 observations before opposition	...	...	...	$8''.947$
27 observations about opposition	...	...	...	8.887
27 observations after opposition	...	...	...	8.783

Of course in this case phase does not enter into the question, nor do these values differ from the mean by quantities much greater than their probable errors. Still the coincidence is remarkable, though I can offer no suggestion to account for such a fact, if real.

We have still to consider the possible systematic errors contained in the result which are common to all the combinations.

* 'Ueber eine Bestimmung der Sonnenparallaxe aus correspondirenden Beobachtungen des Planeten "Flora."' Von Dr. J. G. Galle, Breslau, 1875.

1. *Difference of Habit of Observing Distances E. and W. of the Meridian.*—

This was eliminated by use of the reversing prism, for by its means the measures were all made along the same apparent angle with respect to the observer's eye, and alternately in reverse directions.

2. *Error or Change in the Error of the Adopted Scale-Value.*—This was first eliminated by introducing a term in the equations depending on a supposed unknown error of scale-value ; and second, by an independent discussion in which it was proved that, from the symmetry of the arrangement of the comparison stars, the result was practically identical as to value and accuracy, even if systematic errors of scale-value E. and W. of the meridian are supposed to exist, which exceed any changes of scale-value that are found in practice.

3. *Errors in the Tabular Motion.*—It is shown that, whether the tabular errors are developed in terms of the time from opposition, or whether we derive them from a curve drawn through the various results for $\Delta\alpha$ and $\Delta\delta$, the resulting parallax is the same.

4. *Errors of the Refraction Tables.*—These were so effectually eliminated by the symmetrical arrangement of the comparison stars that even if corrections for refraction had been *entirely omitted*, the resulting parallax would have differed but $0''.014$ from the definitive result, though, of course, the probable error would have been largely increased. No conceivable errors of refraction in the ordinary sense could therefore produce any sensible influence on the result.

If, however, the light of *Mars* is of different mean refrangibility from that of a star, then a systematic error would be created which could not be satisfactorily distinguished from parallax by any possible method of observation or reduction, because, at the altitudes where it is possible to make good observations, the law of the variation of the refraction does not sufficiently differ from the law of the variation of the parallax.

If it is really the case that a difference of refraction does exist for the light of *Mars* as a whole and the light of an average star, it is probable, from its redder hue, that the light of *Mars* would be less refracted than the light of an ordinary star ; and, if this is the case, the parallax ($8''.78$) at which I have arrived, will have to be diminished.

[It would be most interesting and important to have good determinations with a prism of very low dispersion of the mean refrangibility of the visible light of *Mars*, as compared with the mean refrangibility of the visible light of an average star.]

It is, however, in the atmospheric chromatic dispersion of the limbs that I believe we must look for the principal source of systematic error. *A priori* it is probable that heliometer measures are much less subject to such errors than measures with instruments in which a spider line is employed. In observations with the heliometer the image of the star, which is brought into contact with the image of the planet's limb, is coloured by chromatic dispersion of the atmosphere precisely in the same way as the limb of the planet itself, and it is only reasonable to conclude that there should be less tendency to systematic error when this is the case than when oppositely coloured limbs are successively brought into contact with a dark web.

I do not doubt that it is to chromatic dispersion we must attribute the larger values of the $\odot$'s parallax which have been deduced from meridian observations of *Mars*, than from those derived by other methods.

Let us examine the question.

For an image of *Mars* at 65° Z.D. (the Z.D. of *Mars* at $\mathcal{J}$ 1877 for the Observatory of Leiden, from the observations of which, combined with the Melbourne observations, Mr. DOWNING has recently determined a value of the solar parallax), the visible limits of the atmospheric chromatic dispersion amount to $2''.2$ —that is to say, each true upper and lower limb is covered by a fringe of coloured light $2''.2$ broad. Of the two fringed limbs there is no doubt that the one which will appear most extended will be that in which the red (or more glaring colour) is formed beyond the true disk. This is the case in the lower limb, and it will be especially marked in an instrument the field of which is illuminated with white light, by which the violet fringe of the upper limb will be rendered nearly invisible. The natural tendency of the observer will therefore be to cut more deeply into the upper or false violet limb, than into the lower and more visible red false limb: hence, therefore, a systematic tendency to make the Z.D. of the planet too great, and therefore to obtain too large values of the parallax.

So far as the present investigation goes, the results separately derived

from the greater and lesser Zenith Distances are in such satisfactory accord as to show that most probably the results of this investigation are not affected by systematic error due either to difference of refrangibility between the mean light of *Mars* and that of the comparison stars, or to errors due to the atmospheric chromatic dispersion of the limbs. But unfortunately the method which lends itself most powerfully to the determination of this point (viz. the method in which all the observations can be used, and in which it is necessary to assume that the star-places are known) is affected by the larger probable error, and hence the certainty of the freedom of the result from such errors is not so great as is desirable. Subject, however, to a possible small correction due to the sources of error in question, the definitive result of the present investigation is

$$\begin{array}{l} \text{Mean Horizontal Equatorial Parallax of the Sun} = 8.78 \\ \text{With the probable error} \quad \dots \quad \dots \quad \dots \quad \dots \quad \dots \quad \pm 0.012 \end{array}$$

which result, combined with the most modern determination of the value of the Equatorial Radius of the Earth—viz. LISTING'S*—

$$= 3962.79 \text{ miles,}$$

gives for the Mean Distance of the Earth from the Sun

$$93,080,000 \text{ miles.}$$

The value of the parallax thus arrived at is smaller than any which has been generally accepted during the last ten years as the most probable. It is needless here to recount the history of the problem; suffice it to say that the value most generally received, depending as it did upon investigations by several independent methods, has been from $8''.85$ to $8''.86$, whilst a larger value still, approaching $8''.90$, has had the support of some eminent astronomers.

The five methods upon which the chief stress has usually been put are—the Meridian Observations of *Mars*, the Transits of *Venus*, the Parallaxic Inequality of the Moon, the Gravitational Method, and the Velocity of Light

* 'Neue geometrische und dynamische Constanten des Erdkörpers, u.s.w.,' von Johann Benedict Listing. Göttingen, 1878.

combined with the Constant of Aberration, or the Light Equation deduced from eclipses of *Jupiter's* satellites.

1. *Meridian Observations of Mars*.—I have shown a cause of large systematic error common to all these observations, a cause, too, of a kind that must tend very sensibly to give too large values of the parallax from all such observations—viz., the effect of chromatic dispersion of the atmosphere.

2. *Transits of Venus*.—Between the value of the parallax ($8''.58$) deduced by ENCKE, and that ($8''.91$) deduced by STONE from the Transit of *Venus* of 1769, there is a greater difference than between any of the twenty-two independent results of the present discussion. These differences arise entirely from the different interpretation put upon the language of the observers. I do not propose to enter into any discussion of the relative merits of these or other interpretations, it is sufficient to note that such different values have been found from the same observations by astronomers eminent alike for their skill and integrity, and were received with acceptance in their day. I only repeat these well-known facts here to give weight to my argument, that however valuable and however important the opinion of a particular astronomer may be, *any method which depends upon a matter of opinion or judgment, or on the interpretation to be put upon the language of an observer, is not fit to be employed in the solution of a problem like that of the Solar parallax.*

The very different results which have been deduced by AIRY ($8''.75$) and by STONE ($8''.86$) from the very same observations of the Transits of 1874 afford the strongest confirmation of the justice of these remarks.

The results of the present observations of *Mars* are perfectly in accord with reasonable interpretations of the transits of *Venus* both of 1769 and 1874.

3. *The Parallaxic Inequality of the Moon*.—The values of the $\odot$ ' parallax derived from the Parallaxic Inequality of the Moon have, up till this time, depended *entirely* upon the adopted semi-diameter of the Moon, and as that cannot be eliminated from the observations themselves, the method is one that cannot be relied upon for a *determination* of the parallax; for the judg-

ment or opinion of the astronomer must step in, and I think I have already sufficiently shown how dangerous this may be, and how necessary it is to exclude all methods in which such a course must be adopted.

4. *The Gravitational Method.*—The results of LE VERRIER, from the mass of the Earth as derived from the motions of the planets *Venus* and *Mars*, by which a value of $8''.86$ for the $\odot$ parallax was determined by him, appear, according to NEWCOMB, to require a correction of $-0''.03$; and when the complication of the processes through which they have been derived is considered, it is by no means impossible that other necessary points have been overlooked. In 1874 the tabular errors of *Venus*, as determined by the planet's transit across the Sun's disk, amounted to more than $5''$ of arc both in R.A. and Decl., and it is shown in the present investigation that the tabular errors of *Mars* amounted to more than $8''$ in R.A. and to about $3''$ in Declination at the opposition of 1877. LE VERRIER's planetary tables do not therefore possess the accuracy attributed to them by their distinguished author, and the conclusions at which he arrived probably require some further modification, which it is not the object of the present paper to discuss.

5. *The Velocity of Light.*—When FOUCAULT determined the velocity of light in 1862, and deduced from it, combined with STRUVE's constant of aberration, the value $8''.86$ for the solar parallax, the result was hailed as a remarkable confirmation of LE VERRIER's value.

Afterwards, on LE VERRIER's suggestion, CORNU redetermined the velocity of light, and having arranged much more perfect apparatus, he arrived at a result of far greater accuracy, and this result, combined with STRUVE's constant of aberration ($20''.445$), reduced the parallax from $8''.86$ to

$$8''.798.$$

But M. CORNU points out that the velocity of light resulting from his observations, when combined with BRADLEY's constant of aberration ($20''.25$), gives LE VERRIER's value of the parallax.

So far as I know, the following are the best and most generally accepted trustworthy determinations of the Constant of Aberration:—

Ancient Observations.

Date.		
1728	BRADLEY. From the original observations from which he was led to the discovery of aberration	20".25
	BUSCH. From BRADLEY's observations at Kew and Wandsted from 1727 to 1747	20".211
	BESSEL. From BRADLEY's zenith sector observations, 1750-54 (Fundamenta)	20".475

Best Modern Observations.

1831	RICHARDSON. From 2066 observations with TROUGHTON's circle at Greenwich, 1825-28	20".505
	RICHARDSON. From 2053 observations with JONES's circle at the same period	20".502
1842	HENDERSON. By the observations from which he deduced the parallax of α Centauri at the Cape	20".41
1843	PETERS. From meridian transits of Polaris at Dorpat, 1822-38	20".425
1843	LUNDHALL. From meridian observations of declination of Polaris at Dorpat, 1822-38	20".550
1843	STRUVE. From seven stars observed with the prime vertical transit at Pulkowa	20".445
1844	PETERS. From altitudes of the Pole Star with the Pulkowa vertical circle	20".503
1850	MACLEAR. From his observations of direct and reflex observations of the meridian altitude of α Centauri from observations 1842-49	20".53
1851	MACLEAR. From similar observations of β Centauri	20".59
1849	LINDHAGEN. From right ascensions of Polaris at Pulkowa, 1842-44	20".498

The arithmetical mean of all the best modern determinations of this constant gives thus—

$$20''.496.$$

In 1873 NYRÉN published as the conclusions of his paper on the latitude of Pulkowa (*Mem. St. Petersburg Acad.* vol. xix.) a discussion of all the determinations of this constant from the best modern Pulkowa Observations, with the Transit Instrument, the Vertical Circle, and the Prime Vertical Instrument, and deduces the value

$$20''.481 \pm 0''.008,$$

which he considers probably the most reliable determination of this constant which we possess.

These facts would, at least, show that STRUVE's determination should

more probably be increased than diminished. It is difficult, therefore, to understand upon what grounds these beautiful modern investigations should be rejected, and a series made 150 years ago by a now obsolete instrument be adopted instead.* I have included in the quoted results those of HENDERSON and MACLEAR, because they are the only moderately satisfactory determinations of the kind as yet made in the Southern Hemisphere; and so far as their evidence goes, it would appear that there is little or no systematic difference in the constant when determined in either hemisphere, a possibility of which is suggested by VILLARCEAU (*Addition à la Con. du Temps pour 1878*).

Since CORNU's determination of the velocity of light, an apparently still more accurate investigation of this constant has been made by Mr. A. A. MICHELSON, U.S. Navy (*Proceedings of the American Association for the Advancement of Science*, vol. xxvii. 1878). This determination gives a slightly smaller velocity of light than CORNU's, and, combined with STRUVE's constant of aberration, leads to a parallax of

$$8''.81.$$

M. CORNU has ably pointed out the doubt which attaches to this mode of determining the parallax, because of the discordances which arise when, on the one hand, the determined light velocity is combined with DELAMBRE's determination (from the eclipses of *Jupiter's* satellites) of the time (k) required by light to traverse a mean radius of the Earth's orbit, and when, on the other hand, it is combined with STRUVE's determination of the constant of aberration.

But recently GLASENAPP (a young Russian Astronomer), in a Russian

* At the conclusion of his paper on the Telescopic Observations of the Transit of *Venus* (*Monthly Notices*, vol. xxxviii. p. 295), Mr. STONE makes the following statement, by way of confirming the Value of the Solar Parallax ($8''.89$), at which he has arrived:—

“There are also the determinations from the velocity of light which have been lately repeated by M. CORNU, and which, with the accepted value of the coefficient of Aberration, lead to a value of $8''.86$.”

The value of the coefficient of Aberration which, when combined with the velocity of light determined by CORNU, corresponds with $8''.86$ for the Solar Parallax, is $20''.25$. It would, therefore, appear that Mr. STONE considers BRADLEY's original result to be “the accepted value of the coefficient of Aberration.” This view is hardly supported by the modern evidence above quoted.

thesis, has discussed the eclipses of the first satellite of *Jupiter* from 1848 to 1873, and he obtains

$$\begin{array}{ll} \text{Instead of DELAMBRE'S value of } k = 493\cdot 2^s & \\ \text{The very different value} & k = 500\cdot 8 \pm 1\cdot 02. \end{array}$$

Mr. TODD, of the American Nautical Almanac office, has done good service in calling attention to these results of GLASENAPP, and to the recent determinations of the velocity of light (*American Journal of Science*, vol. xix. p. 62).

He remarks with much reason—

“Only two astronomically precise determinations of this interval (k) have ever been made ; the first by DELAMBRE in the early part of the present century, from a discussion of an immense mass of eclipses of the satellites of *Jupiter*, comprising observations from 1662 to 1802 ; the second by GLASENAPP, in a Russian thesis, in which there are discussed the observations of the first satellite of *Jupiter* from 1848 to 1873. . . .

“It is quite impossible to judge with certainty how these two widely discordant results should be combined. The former determination rests on a much greater number of observations than the latter ; *but it is difficult to form a just estimate of the worth of an average last century observation of an eclipse of a satellite of Jupiter.* And, moreover, *astronomers have no means of knowing the process which led the distinguished French astronomer to his result—which was adopted in his own tables of the satellites, and which was adopted by DAMOISEAU in his Tables Ecliptiques, published in 1836. The latter determination rests upon a mass of observations of definite excellence, which have been discussed after the modern fashion.*”

Mr. TODD then combines DELAMBRE'S value with GLASENAPP'S, by giving double weight to the latter. Having regard to those portions of Mr. TODD'S remarks which I have printed in italics, I think that the general opinion of astronomers would be that the latter value is the only one which can be seriously considered and taken into account in a discussion of the most probable value of the parallax.

GLASENAPP'S determination of k , combined with MICHELSON'S value of the velocity of light, gives

$$8\cdot 76$$

for the solar parallax, and STRUVE's constant of aberration combined with the same determination of the velocity of light gives

$$8''.81.$$

The mean of these two results is

$$8''.785,$$

which may be supposed to represent the best modern determination of the velocity of light combined on the one hand with the best single determination of the constant of aberration, and on the other hand with the best determination of the value of k ; and this result is in perfect harmony with the result from the observations of *Mars* discussed in the present *Memoir*, viz.—

$$8''.78.$$

It is also easy to see that if, instead of STRUVE's constant of aberration, we adopt the mean value of the best modern determinations of this constant, we get with MICHELSON's light velocity $8''.78$ for the solar parallax. Also, if we suppose GLASENAPP's determination of k to be too great by one second (that is to be changed in the direction of DELAMBRE's value by the amount of the probable error of its determination), we again obtain $8''.78$ for the solar parallax.

But nothing is easier and nothing less profitable than to form such coincidences by the variation of weights or the adoption of particular results; the question rather is what are the methods that should be followed in the future to settle still more definitely and precisely this great problem.

I venture to submit to the opinion of competent judges the following proposals for this purpose :—

- (1) A satisfactory re-determination of the constant of aberration.
 - (2) Numerous and long-sustained series of observations of the eclipses of *Jupiter's* satellite, probably aided by photometric determinations according to Professor PICKERING's plan.
 - (3) A new theory of *Jupiter's* satellites, and a comparison of this theory
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with the observations most suitable for determining the light equation.

- (4) Above all, a really definitive instrumental determination of the parallax by some method not subject to the slightest suspicion of liability to systematic error.

As to the *first* of these, STRUVE's determination of the constant of aberration, by the Prime Vertical Transit at Pulkowa, is probably one of the most refined investigations in astronomy, but in view of M. VILLARCEAU's remarks in his paper "On the Constant of aberration and the velocity of light considered in relation to the absolute movement of the Solar System," it is desirable that STRUVE's work should be repeated in the Southern Hemisphere.

With regard to the *fourth* suggestion which I have ventured to make, I believe it is possible to prove that there is only one known method which promises to fulfil the required conditions of the problem. Thus—

First, we are tied down to the employment of the method of the diurnal parallax, because only by this method can possible personal errors be certainly avoided.

Second, we must adopt the heliometer method because it is now proved to be much more accurate than the methods of transits.

Third, we must adopt the method of minor planets, because with them, and them only (since their images are indistinguishable from stars), can we be *absolutely certain* that the effects of atmospheric chromatic dispersion will not vitiate the results.

It was my intention to propose such a scheme, when my appointment to the Royal Observatory, Cape of Good Hope, obliged me to defer its execution. My desire was to obtain a heliometer of the optical power necessary for observing satisfactorily stars of the 10th magnitude, and to convey it to some equatorial station for a sufficient time to observe the oppositions of four or five minor planets.

If astronomers desire to obtain a really satisfactory and definitive settlement of the Sun's parallax within a reasonable time, a portable heliometer of 6 or 7 inches aperture (which could deal with the faintest stars of the

Durchmusterung) should be constructed, and its constants be determined with the greatest care. If such an instrument were ready, it might be conveyed from time to time, when exceptionally favourable oppositions offered (as, for example, those of *Victoria* and *Sappho* in 1882), to such stations as would be most favourably situated for the particular opposition in question.

In this way, without undue loss of time, advantage might be taken of the most favourable opportunities, and in course of a few years such a series of determinations would be obtained as would set at rest this most important and difficult question. The total cost would be but trifling compared with that of a Transit of *Venus* Expedition, whilst it is certain that the results would be of much greater value. For my own part I would gladly take such an instrument in charge, and perform the necessary observations at the different stations from time to time, when favourable opportunities occur.

For planets of S. declination the Seychelles Islands offer, perhaps, the best station, and for planets of N. declination Ceylon is very suitable.

In the preceding work I have endeavoured—

- (1) To arrive at the most reliable value of the $\odot$'s parallax from my own observations at Ascension Island, and to show what possible errors that result may be liable to.
- (2) To indicate possible errors to which other determinations of the same constant are liable, and to compare the results of such of these as seem least liable to systematic error, with the result of my own observations.
- (3) To prove to demonstration the means which should be taken, in the present state of our experience, if an independent and entirely satisfactory instrumental determination of the parallax is to be made.

If in the judgment of the Society I have been successful in these endeavours, I shall not regret the anxiety and labour which this work has involved.

*Royal Observatory,
Cape of Good Hope:
May, 1880.*

Postscript.

The following Table gives the errors of the *Nautical Almanac* places of *Mars* (LE VERRIER'S Tables) as derived from the preceding separate observations.

The Preliminary Correction is equivalent to an error of $+2''.45$ in Heliocentric Longitude.

Date, 1877.	Right Ascensions.			Declinations.		
	Preliminary Correction.	$\Delta\alpha$ from Observation.	Correction of LE VERRIER'S Tables to Observed R.A.	Preliminary Correction.	$\Delta\delta$ from Observation.	Correction of LE VERRIER'S Tables to Observed Dec.
d	"	"	"	"	"	"
July 31.62	+6.16	-0.21	+5.95	+3.10	+0.11	+3.21
Aug. 5.58	+6.58	- .34	+6.24	+3.26	+ .14	+3.40
11.61	+7.12	- .05	+7.07	+3.41	- .26	+3.15
14.35	+7.34	- .14	+7.20	+3.43	- .39	+3.04
15.57	+7.45	- .48	+6.97	+3.43	- .69	+2.74
16.57	+7.54	+ .25	+7.79	+3.43	- .48	+2.95
19.52	+7.81	+ .84	+8.65	+3.43	- .63	+2.80
24.59	+8.19	+ .10	+8.29	+3.44	- .50	+2.94
29.54	+8.46	- .02	+8.44			
Sept. 2.47				+3.46	- .65	+2.81
4.36				+3.46	- .54	+2.92
4.42				+3.46	- .58	+2.88
5.59	+8.56	+ .73	+9.29	+3.46	- .71	+2.75
6.51	+8.55	+ .80	+9.35			
7.51	+8.53	+ .47	+9.00			
8.47	+8.50	+ .16	+8.66			
8.57	+8.50	- .18	+8.32	+3.43	- .80	+2.63
9.60						
10.26	+8.44	+ .23	+8.67	+3.43	- .70	+2.73
10.44						
10.90	+8.41	+ .21	+8.62			
26.49	+7.45	- .17	+7.28	+2.86	- .21	+2.65
Oct. 3.49	+6.85	+ .74	+7.59	+2.52	+ .09	+2.61